Problem Set 7

1. Prove that $z_8 \oplus z_2$ is not isomorphic to $z_4 \oplus z_4$.

2. Prove that the group of complex numbers under addition is isomorphic to $\mathbb{R} \oplus \mathbb{R}$.

3. In $z_{40} \oplus z_{30}$, find two subgroups of order 12.

4. Find a subgroup of $z_{12} \oplus z_4 \oplus z_{15}$ that has order 9.

5. Find an isomorphism from z_{12} to $z_4 \oplus z_3$.

6. Suppose that ϕ is an isomorphism from $z_3 \oplus z_5$ to z_{15} and $\phi(2,3) = 2$. Find an element in

    $z_3 \oplus z_5$ that maps to 1.

7. Let $H = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \mid a,b,d \in \mathbb{R}, ad \neq 0 \right\}$. Is H a normal subgroup of $GL(2,\mathbb{R})$?

8. Prove that a factor group of a cyclic group is cylic.

9. Prove that a factor group of an abelian group is abelian.

10. What is the order of the element $14 + \langle 8 \rangle$ in the factor group $Z_{24} / \langle 8 \rangle$?