MAS 4300: Abstract Algebra Broward College

Problem Set 6

Directions: Work all of the following problems.

1. Prove or Disprove that $U(20)$ and $U(24)$ are isomorphic.

2. Let $G = \{a + b\sqrt{2} \mid a, b \text{ rational}\}$ and $H = \left\{ \begin{bmatrix} a & 2b \\ b & a \end{bmatrix} \mid a, b \text{ rational} \right\}$. Show that G and H are isomorphic under addition. Does your isomorphism preserve multiplication as well?

3. Let $R^n = \{(a_1, a_2, ..., a_{n-1}, a_n) \mid a_i \in R\}$. Show that the mapping $\phi : (a_1, a_2, ..., a_n) \to (-a_1, -a_2, ..., -a_n)$ is an automorphism of the group R^n under componentwise addition. Describe the action of the *inversion* , ϕ , geometrically.

4. Let $G = \{0, \pm 2, \pm 4, \pm 6, ...\}$ and $H = \{0, \pm 3, \pm 6, \pm 9, ...\}$. Show that G and H are isomorphic groups under addition. Generalize the case when $G = \langle m \rangle$ and $H = \langle n \rangle$.

5. Show that every automorphism ϕ of the rational numbers Q under addition to itself has the form $\phi(x) = x\phi(1)$.

6. Let n be a positive integer. Let $H = \{0, \pm n, \pm 2n, \pm 3n, ...\}$. Find all left cosets of H in Z. How many are there?

7. Suppose that a has order 15. Find all of the left cosets of $\langle a^5 \rangle$ in $\langle a \rangle$.

8. Let C^* be the group of nonzero complex numbers under multiplication and let $H = \{a + bi \in C^* \mid a^2 + b^2 = 1\}$. Give a geometric description of the coset $(c + di)H$.

9. Suppose that K is a proper subgroup of H and H is a proper subgroup of G. If $|K| = 42$ and $|G| = 420$, what are the possible orders of H?

10. Prove that the order of $U(n)$ is even when $n > 2$.

11. Suppose $|G| = 8$. Show that G must have an element of order 2.