MAS 4300: Abstract Algebra Broward College

Problem Set 5

Directions: Work all of the following problems.

1. How many elements of order 5 are in s_7? You must justify your answer.

2. Prove that (1234) is not the product of 3-cycles.

3. Let $\beta = (123)(145)$. Write β^{99} in disjoint cycle form.

4. Let $\beta = (1,3,5,7,9,8,6)(2,4,10)$. What is the smallest positive integer n for which $\beta^n = \beta^{-5}$? You must justify your work.

5.
   a. Let $H = \{\beta \in S_5 \mid \beta(1) = 1 \text{ and } \beta(3) = 3\}$. Prove that H is a subgroup of s_5.
   b. How many elements are in H? Is your argument valid in S_n for any n? How many elements are in H in this case?

6. Find an isomorphism from the group of integers under addition to the group of even integers under addition.

7. Let R^+ be the group of positive real numbers under multiplication. Show that the mapping $\phi(x) = \sqrt{x}$ is an automorphism of R^+.

8. Show that $U(8)$ is not isomorphic to $U(10)$.

9. Show that $U(8)$ is isomorphic to $U(12)$.

(Cont.)

10. Show that the mapping $a \to \log_{10} a$ is an isomorphism from R^+ under multiplication to R under addition.

11. Let G be a group. Prove that the mapping $\alpha(g) = g^{-1}$ for all $g \in G$ is an automorphism if and only if G is abelian.

12. Suppose that $\phi: Z_{50} \to Z_{50}$ is an automorphism with $\phi(11) = 13$. Find a formula for $\phi(x)$.