MAS 4300: Abstract Algebra Broward College

Homework Assignment # 6 - Solutions

Name: ________________________________ **I.D. #:** ______________________________

Directions: Work all of the following problems.

1. Write each of the following permutations as the product of disjoint cycles.

   a. $(1235)(413)=(15)(234)$

   b. $(13256)(23)(46512)=(124)(35)$

   c. $(12)(13)(23)(142)=(1423)$

QED

2. Find the order of $(124)(357869)$

   Solution:

   Order is the LCM of the lengths of each disjoint cycle. So,
   $lcm(6,3)=6$.

QED

3. What is the order of $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 4 & 6 & 3 \end{bmatrix}$.

   Solution:

   $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 4 & 6 & 3 \end{bmatrix}=(12)(356)$. So, the order is the LCD of the disjoint cycles. So,

   $lcm(2,3)=6$. Thus, the order of $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 4 & 6 & 3 \end{bmatrix}$ is 6 .

QED

4. Find the inverse of:

a. $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 4 & 6 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 6 & 4 & 3 & 5 \end{bmatrix}$

b. $(213546)^{-1} = (645312)$

c. $(a_1 a_2 a_3 \ldots . a_n)^{-1} = (a_n a_{n-1} a_{n-2} \ldots . a_3 a_2 a_1)$

QED

5. Show that a function from a finite set S onto itself is one-to-one if and only if it is onto. Is this true if S were infinite?

Solution:

Proof**:**

Say, $S = \{s_1 s_2 \ldots . s_n\}$ and ϕ is one-to-one from S to S. Then $\phi(s_1), \phi(s_2), \phi(s_3) \ldots, \phi(s_n)$ are all distinct and all in S. So, $\phi(S) = S$. So, ϕ is onto.

On the contrary, if $\phi(s_i) = \phi(s_j)$ for some $i \neq j$. Then $\phi(S)$ has at most $n-1$ members. Since this clearly cannot happen, we see that the mapping is one-to-one.

The mapping from Z to Z that takes x to 2x is one-to-one, but not onto. So, this property is not true if S were infinite.

QED