MAS 4300: Abstract Algebra Broward College

Problem Set 3

Directions: Work all of the following problems.

1. Find all the generators of $Z_6, Z_8,$ and Z_{20}.

2. Suppose that a cyclic group G has exactly three subgroups: G itself, $\{e\}$, and a subgroup of order 7. What is $|G|$? What can you say if 7 were replaced by p, where p is a prime?

3. Prove that Z_n has an even number of generators if $n > 2$.

4. Show $Z_{2^{2002}}$ has no subgroup of order 3^k for any $k \geq 1$.

5. Let $H = \left\{ \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \mid n \in Z \right\}$. Show that H is a cyclic subgroup of $GL(2, R)$.

   (Hint: You must first show that H is a subgroup and then show H cyclic.)