MAS 4300: Abstract Algebra Broward College

Problem Set 2

1. Let x and y be elements of order 2 in any group. Prove that if $t = xy$ then $tx = xt^{-1}$.

2. Suppose $G = \langle a \rangle$. Show $G = \langle a^{-1} \rangle$

3. Find all generators of Z_6 and Z_8.

4. Show in $G = (Z_n, +)$, for any $x \in G$, $|x| = |n - x|$.

5. Suppose G a group and $a \in G$. Show that if a has infinite order in G, then $a^m \neq a^n$ whenever $m \neq n$.

6. Let G be a group and let $a \in G$. Prove that $C(a) = C(a^{-1})$.

7. Prove that and abelian group with two elements of order 2 must have a subgroup of order 4.

8. Prove that if G an Abelian Group with identity e, then $H = \{x \mid x \in G \text{ and } x^n = e\} \leq G$

   (Recall, in the special case when $n = 2$, we did not require abelian!)

9. Find the center of D_4, the dihedral group of order 8. Justify your answer.