Broward College MAS 4300: Abstract Algebra

Problem Set 1

1. Show that $GL(2,R) = \{A \mid A$ is a square 2 x 2 matrix and $\text{Det}(A) \neq 0\}$ is a non-abelian group by exhibiting a pair of matrices A and B in $GL(2,R)$ such that $AB \neq BA$.
2. Find the inverse of A = $\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix}$ in $GL(2,Z_{11})$.
3. For any integer $n > 2$, show that there are at least two elements in the group $U(n) = \{x \mid x < n$ and $(x,n) = 1\}$ that satisfy $x^2 = 1$ (i.e. at least two elements have their inverses being themselves).

4. Let G be a group with the property that whenever $a,b,c \in G$ and $ab = ca$, then $b = c$. Prove that G is abelian. (i.e. prove that cross cancellation in a group implies commutative!)

5. Prove: If G a group, $a,b \in G$, and $(ab)^2 = a^2b^2$, then G abelian.

6. Consider the group $G = (Z_{10}, +_{10})$. What is $|G|$ (i.e. the order of G). Also, Find $|3|$ (the order of the element 3 in G), and $|4|$ (the order of 4 in G).