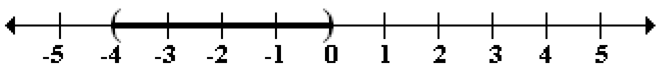
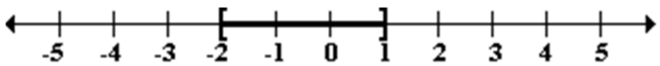
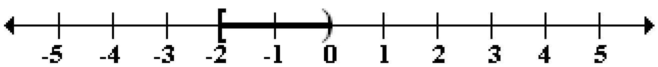
Union and Intersection of Intervals

p.202 #16: Find $(-4,0) \cap [-2,1]$.

$(-4,0)$: 

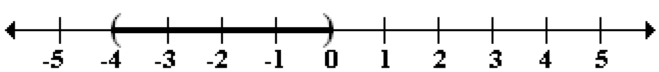
$[-2,1]$: 

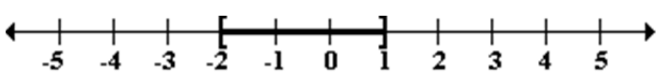
To find the intersection, take the portion of the number line that the two graphs have in common.

Answer: 

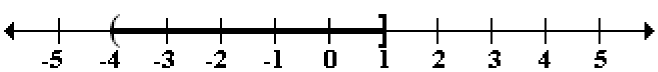
; $[-2,0)$

p.202 #18: Find $(-4,0) \cup [-2,1]$.

$(-4,0)$: 

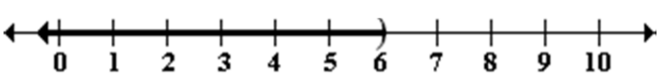
$[-2,1]$: 

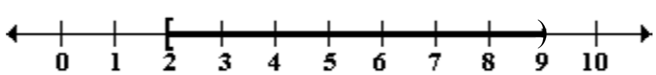
To find the union, take the portion of the number line that is in either graph.

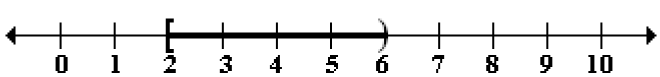
Answer: 

; $(-4,1]$

p.202 #20: Find $(-\infty,6) \cap [2,9)$.

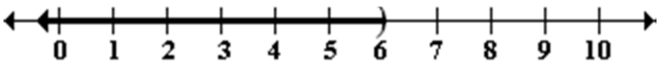
$(-\infty,6)$: 

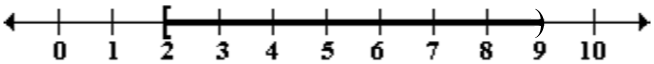
$[2,9)$: 

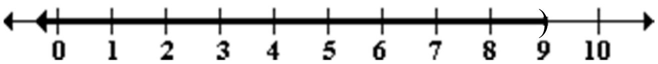
Answer: 

; $[2,6)$

p.202 #22: Find $(-\infty,6) \cup [2,9)$.

$(-\infty,6)$: 

$[2,9)$: 

Answer: 

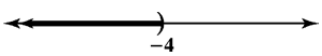
; $\boxed{(-\infty,9)}$

Linear Inequalities

Example: Solve $-7x > 28$.

Important: When you multiply or divide both sides of an inequality by a negative number, you must reverse the inequality symbol.

Divide both sides by -7 to obtain $\frac{-7x}{-7} < \frac{28}{-7}$, which simplifies to $x < -4$.



The solution set in interval notation is $\boxed{(-\infty,-4)}$.

p.203 #42: Solve $\frac{3x}{10} + 1 \geq \frac{1}{5} - \frac{x}{10}$.

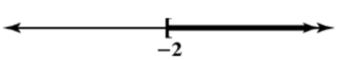
Multiply both sides by 10 to clear the fractions to obtain $\mathbf{10} \cdot \frac{3x}{10} + \mathbf{10} \cdot 1 \geq \mathbf{10} \cdot \frac{1}{5} - \mathbf{10} \cdot \frac{x}{10}$.

Simplify to obtain $3x + 10 \geq 2 - x$.

Add x to both sides to obtain $4x + 10 \geq 2$.

Subtract 10 from both sides to obtain $4x \geq -8$.

Divide both sides by 4 to obtain $x \geq -2$.



The solution set in interval notation is $\boxed{[-2,\infty)}$.

p.203 #48: Solve $3(x - 8) - 2(10 - x) > 5(x - 1)$.

Distribute to obtain $3x - 24 - 20 + 2x > 5x - 5$.

Combine like terms to obtain: $5x - 44 > 5x - 5$.

Subtract 5x from both sides to obtain $-44 > -5$.

Since $-44 > -5$ is always a false statement, that means there are no real numbers that make the original inequality true. Thus, the solution set is $\boxed{\varnothing}$ (the empty set).

Important: If we had obtained $-44 < -5$, which is always a true statement, that means every real number makes the original inequality true. Thus, the solution set in interval notation would be $\boxed{(-\infty,\infty)}$.

Compound Inequalities

Compound inequalities involve the word "or" or the word "and".

p.203 #56: Solve $3 \le 4x - 3 < 19$.

Note that the given inequality is called a 3 part inequality.

We want to get x by itself in the middle part of the inequality.

Add 3 to each part of the inequality to obtain $\begin{array}{rcrcr} 3 & \le & 4x-3 & < & 19 \\ +3 & & +3 & & +3 \end{array}$, which simplifies to $6 \le 4x < 22$.

Divide each part by 4 to obtain $\frac{6}{4} \le \frac{4x}{4} < \frac{22}{4}$, which simplifies to $\frac{3}{2} \le x < \frac{11}{2}$.

This means that $x \ge \frac{3}{2}$ **and** $x < \frac{11}{2}$.

The solution set in interval notation is $\boxed{\left[\frac{3}{2},\frac{11}{2}\right)}$.