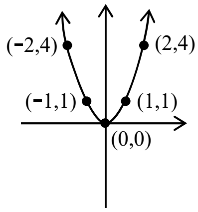
MAC 1105 - Transformations of Functions - Section 2.5

Mr. Levy

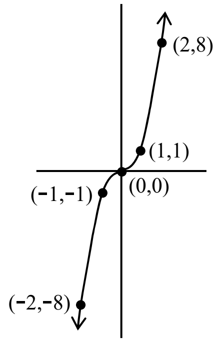
Basic Functions

$y = x^2$



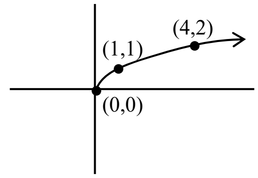
$D = (-\infty,\infty)$
$R = [0,\infty)$

$y = x^3$



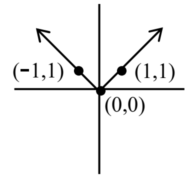
$D = (-\infty,\infty)$
$R = (-\infty,\infty)$

$y = \sqrt{x}$



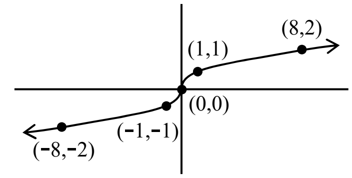
$D = [0,\infty)$
$R = [0,\infty)$

$y = |x|$



$D = (-\infty,\infty)$
$R = [0,\infty)$

$y = \sqrt[3]{x}$



$D = (-\infty,\infty)$
$R = (-\infty,\infty)$

Transformations of Graphs

Equation	Effect on the basic graph $y = f(x)$	Remarks
1. $y = f(x) + c$, $c > 0$	Shift *upward* c units.	
2. $y = f(x) - c$, $c > 0$	Shift *downward* c units.	
3. $y = f(x + c)$, $c > 0$	Shift to the *left* c units.	
4. $y = f(x - c)$, $c > 0$	Shift to the *right* c units.	
5. $y = -f(x)$	*Reflect* through the x-axis.	To reflect a graph through the x-axis, change the sign of each y-coordinate.
6. $y = f(-x)$	*Reflect* through the y-axis.	To reflect a graph through the y-axis, change the sign of each x-coordinate.
7. $y = c \cdot f(x)$	Multiply each y-coordinate by c.	If $0 < c < 1$, the graph of $y = f(x)$ is said to be vertically shrunk. If $c > 1$, the graph of $y = f(x)$ is said to be vertically stretched.

Multi-Transformations of Graphs

1. $y = a(x - h)^2 + k \Rightarrow$ vertex = (h,k)
2. $y = a(x - h)^3 + k \Rightarrow$ inflection point = (h,k)
3. $y = a\sqrt{x - h} + k \Rightarrow$ starting point = (h,k)
4. $y = a|x - h| + k \Rightarrow$ vertex = (h,k)
5. $y = a\sqrt[3]{x - h} + k \Rightarrow$ inflection point = (h,k)

Order of Transformations

1. Horizontal shifting
2. Stretching or shrinking
3. Reflecting
4. Vertical shifting

Remark: A negative value for "a" will cause a reflection through the x-axis.

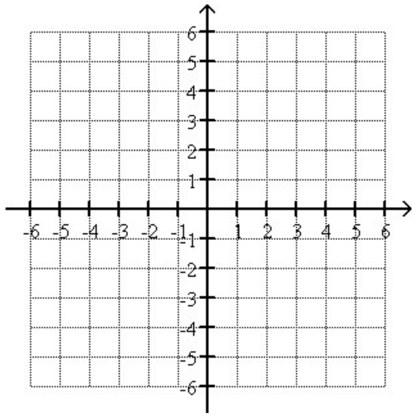
Examples: Graph each function.

1. $y = |x| + 2$

Basic graph:

y = _______

Final answer:

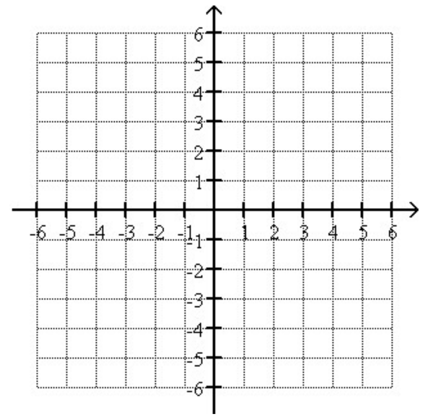


2. $y = (x - 4)^3$

Basic graph:

y = _______

Final answer:

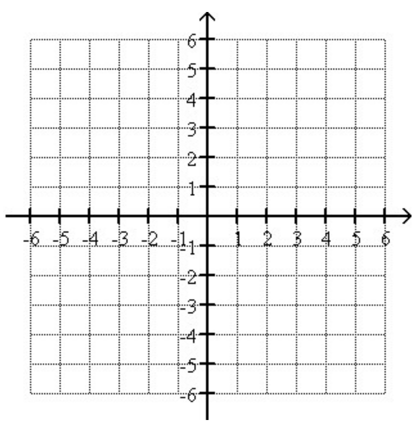


3. $y = -\sqrt{x}$

Basic graph:

y = _______

Final answer:

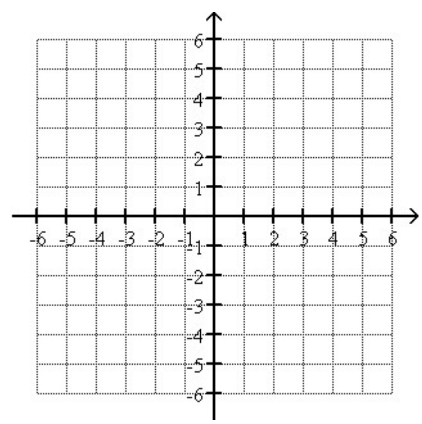


4. $y = \sqrt{-x}$

Basic graph:

y = _______

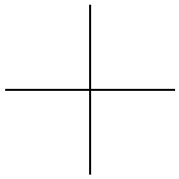
Final answer:



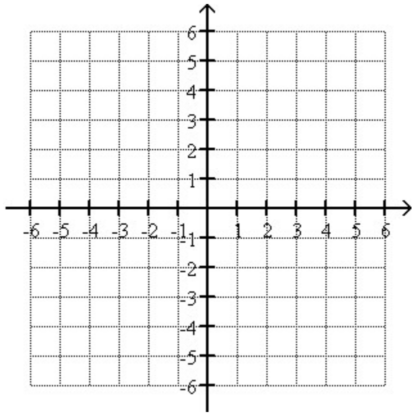
5. $y = -\frac{1}{3}x^2$

Basic graph:

$y =$ _______



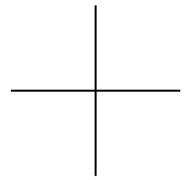
Final answer:



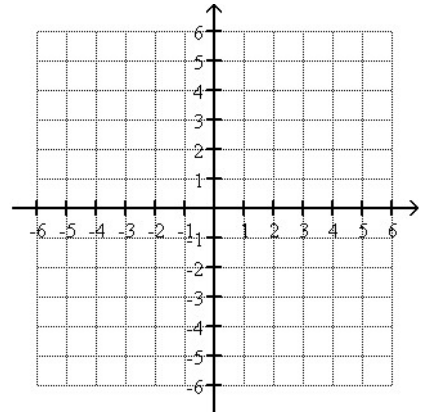
6. $y = |x - 2| - 3$ (multi-transformation)

Basic graph:

$y =$ _______



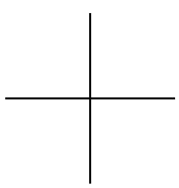
Final answer:



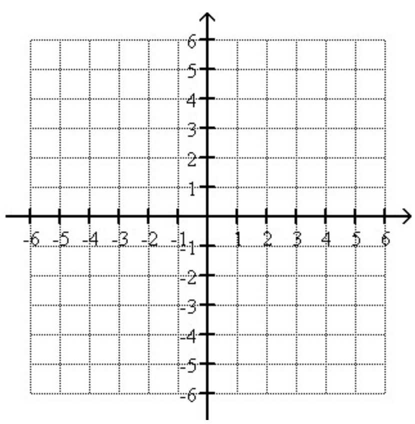
7. $y = 3\sqrt{x+1}$ (multi-transformation)

Basic graph:

$y =$ _______



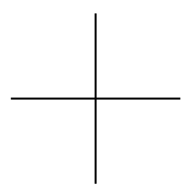
Final answer:



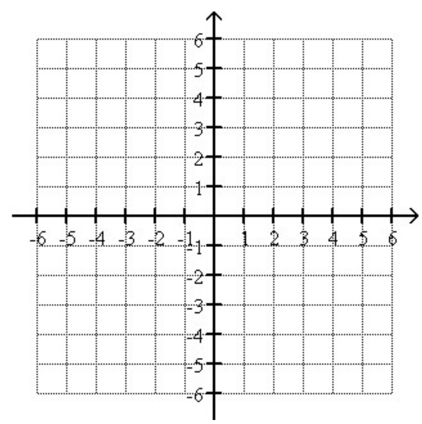
8. $y = -2x^3 + 1$ (multi-transformation)

Basic graph:

$y =$ _______



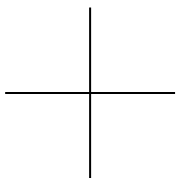
Final answer:

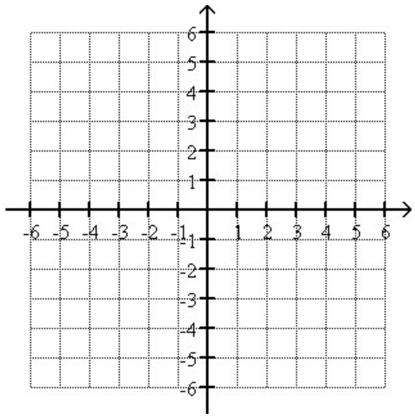


9. $y = \frac{1}{2}\sqrt[3]{x+3}$ (multi-transformation)

Basic graph: Final answer:

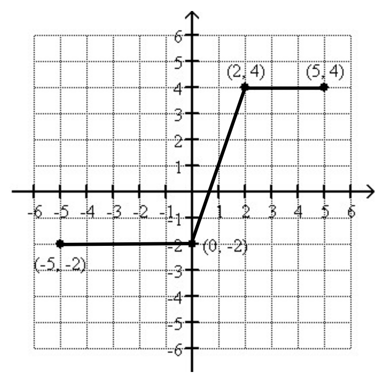
y = _______



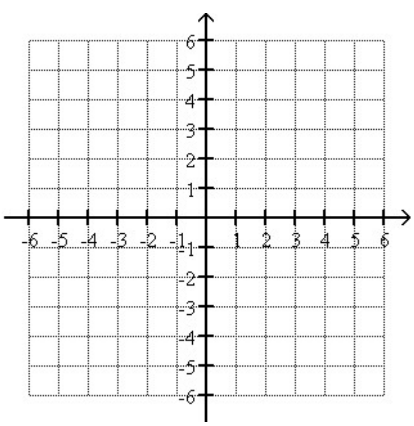


Graphing Transformations of Unknown Functions

Use the graph of $y = f(x)$ shown to the right to graph each function g.



1. $g(x) = 2f(x+1) - 2$



2. $g(x) = 3f(-x)$