4/5/2018 LECTURE NOTES

HARMONIC FORCES

$\frown = rw$

$a = r^2 w$ ANGULAR MOTION $w = \frac{2\pi}{T}$ T = PERIOD

ORBITAL PERIOD

$F = \frac{GMm}{r^2}$ GRAVITATIONAL TO ORBIT WE MUST EQUATE THE FORCES

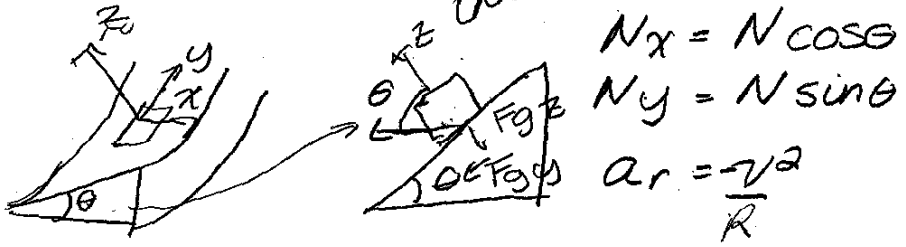
$F = \frac{mv^2}{r}$ CENTRIPETAL

$\frac{mv^2}{r} = \frac{GMm}{r^2 r} \Rightarrow v^2 = \frac{GM}{r} \Rightarrow \frac{r^2}{r}\left(\frac{2\pi}{T}\right)^2 = \frac{GM}{r r^2}$

$\left(\frac{2\pi}{T}\right)^2 = \frac{GM}{r^3} \Rightarrow \frac{T^2}{(2\pi)^2} = \frac{r^3}{GM} \Rightarrow \sqrt{T^2} = \sqrt{\frac{(2\pi)^2 r^3}{GM}}$

$$\boxed{T = 2\pi\sqrt{\frac{r^3}{GM}}}$$

RACE TRACK PERIOD



$N_x = N\cos\theta$

$N_y = N\sin\theta$

$a_r = \frac{v^2}{R}$

$\Sigma F_r = -N\sin\theta = m a_r$

$\frac{-N\sin\theta}{-N} = \frac{-mv^2}{-Nr}$

$\sin\theta = \frac{mv^2}{Nr}$

$\Sigma F_z = N\cos\theta - mg = 0$

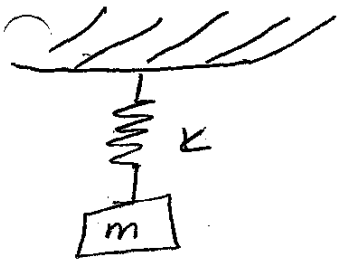
$N\cos\theta - mg = 0 \Rightarrow \frac{N\cos\theta}{N} = \frac{mg}{N} \Rightarrow \cos\theta = \frac{mg}{N}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{mv^2}{Nr}\left(\frac{N}{mg}\right) = \frac{v^2}{rg}$ rg

$rg\tan\theta = v^2 \Rightarrow r^2\left(\frac{2\pi}{T}\right)^2 = rg\tan\theta \Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{g\tan\theta}{r}$

$$T = 2\pi\sqrt{\frac{r}{g\tan\theta}}$$

SPRING PERIOD (OSCILLATION)



$F = -kx$ k = SPRING CONSTANT = kg/s²

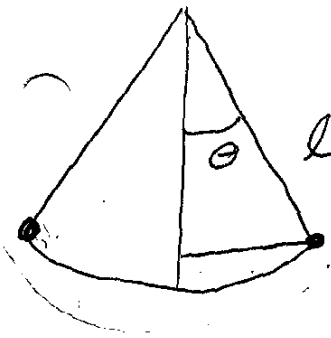
$F = ma$ $a = r^2\omega$; $r = x$

$ma = -kx \Rightarrow +mx\omega^2 = +kx \Rightarrow \frac{m\omega^2}{m} = \frac{k}{m}$

$\omega^2 = \frac{k}{m} \Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{k}{m} \Rightarrow \sqrt{\frac{T^2}{(2\pi)^2}} = \sqrt{\frac{m}{k}}$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

PENDULUM PERIOD (OSCILLATION)



$\sin\theta = \frac{l}{\sqrt{l^2 + l^2\theta^2}}$ $\quad l = r$

$ma = mg - mg\sin\theta$

$a = g - g\sin\theta$

$rw^2 = g - g\sin\theta$

$rw^2 = g(1 - \sin\theta)$

$\frac{rw^2}{l} = \frac{g(1-\sin\theta)}{l}$

$w^2 = \frac{g(1-\sin\theta)}{l}$

$\left(\frac{2\pi}{T}\right)^2 = \frac{g(1-\sin\theta)}{l}$

$$T = 2\pi\sqrt{\frac{l}{g(1-\sin\theta)}}$$ AND FOR $\lim_{\theta \to 0}$ $$T = 2\pi\sqrt{\frac{l}{g}}$$

EXAMPLE 1

A DRAGON SPACE CRAFT IS ORBITING AT 410.0km ON ITS WAY TO THE INTERNATIONAL SPACE STATION. WHAT IS IT'S ORBITAL PERIOD? (EARTH RADIUS 6,378km; $G = 6.674 \times 10^{-11} m^3/kgs^2$; EARTH MASS = $5.972 \times 10^{24} kg$).

HOW STRONG IS THE SPRING IN ORDER TO STOP THE CAPSULE IN 0.250 METER AT THE STATION? ($V = 7.852$ km/s; $V_0 = 7.663$ km/s; $m_{CAPSULE} = 4{,}200kg$).

$$T = 2\pi\sqrt{\frac{r^3}{Gm}} = 2\pi\sqrt{\frac{(6.788\times10^{6}\,m)^3}{(6.674\times10^{-11}\,m^3/kg\,s^2)(5.972\times10^{24}\,kg)}}$$

$$= 5,505\,s\left(\frac{1\,HOUR}{3,600\,s}\right) = \boxed{1.529\,HOUR}$$

$$\frac{ma}{x} = \frac{kx}{x}\;;\quad a = \frac{v^2 - v_0^2}{2d}\;;\quad d = x$$

$$\frac{ma}{d} = k \Rightarrow k = \frac{m}{d}\left(\frac{v^2 - v_0^2}{2d}\right)$$

$$k = (4,200\,kg)\left(\frac{(7,852\,m/s)^2 - (7,603\,m/s)^2}{2(0.250\,m)^2}\right) = \boxed{9.853\times10^{10}\,kg/s^2}$$

EXAMPLE 2

AN INDY CAR (1,952 kg) IS ON A STEEP BANK $\theta = 35.00°$, WHAT IS THE PERIOD OF THE CAR IN A 667.2 m TRACK? HOW LONG WOULD LENGTH WOULD BE FOR PENDULM TO TRACK THIS ON A CLOCK?

$$T = 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{(667.2\,m)}{(9.810\,m/s^2)\tan(35.00°)}} = 61.93\,s$$

$$\frac{T}{2\pi} = \frac{2\pi}{2\pi}\sqrt{\frac{l}{g}} \Rightarrow \left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{l}{g}}\right)^2 \Rightarrow g\left(\frac{T}{2\pi}\right)^2 = \frac{l}{g}\,g$$

$$l = g\left(\frac{T}{2\pi}\right)^2 = (9.810\,m/s^2)\left(\frac{61.93\,s}{2\pi}\right)^2 = \boxed{953.2\,m}$$