4-24-2018 LECTURE NOTES

ENERGY

$T = KE + PE$ TOTAL ENERGY

$KE = \frac{1}{2} m v^2$

$W = \int \vec{F} \cdot d\vec{r}$ INTEGRATION

GRAVITY $\vec{F} = mg\hat{k}$

$W = -\int_0^h mg \cdot dz = -mgz \Big|_0^h = -mgh$

$\cancel{2}\,\frac{1}{2} \cancel{m} v^2 = \cancel{2}\,\cancel{m} g h \Rightarrow \sqrt{v^2} = \sqrt{2gh} \Rightarrow \boxed{v = \sqrt{2gh}}$

UNIVERSAL GRAVITY $\vec{F} = -\frac{GMm}{r^2}\hat{r}$

$W = -\int_r^{\infty} \frac{GMm}{r^2}\, dr = -\frac{GMm}{r}\Big|_r^{\infty} = -\frac{GMm}{r}$

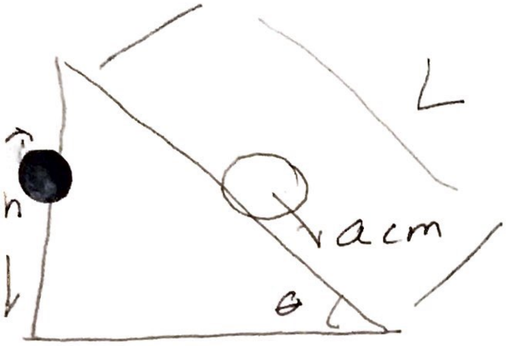
$\cancel{2}\,\frac{1}{2} \cancel{m} v^2 = \cancel{2}\,\frac{GM\cancel{m}}{r} \Rightarrow \sqrt{v^2} = \sqrt{\frac{2GM}{r}} \Rightarrow \boxed{v = \sqrt{\frac{2GM}{r}}}$

SPRING $\vec{F} = kx\hat{i}$

$W = -\int_0^x kx\, dx = -\frac{1}{2}kx^2 \Big|_0^x = -\frac{1}{2}kx^2$

$\frac{\cancel{\frac{1}{2}} m v^2}{m} = \frac{\cancel{\frac{1}{2}} k x^2}{m} \Rightarrow \sqrt{v^2} = \sqrt{\frac{kx^2}{m}} \Rightarrow \boxed{v = \sqrt{\frac{kx^2}{m}}}$

INCLINED PLANES



WITH A ROLLING ROD

$F_F = \mu N$

$F_F = \mu_K Mg\cos\theta$

$\Sigma F_x = Mg\sin\theta - F_f = M a_{cm}$

$\Sigma F_y = N - Mg\cos\theta = 0 \Rightarrow N = Mg\cos\theta$

$\tau = F \cdot r \Rightarrow \text{FRICTION} = F_F R = I_{cm}\alpha \; ; \; \alpha = \frac{a_{cm}}{R}$

$I = \frac{1}{2}MR^2$

$\mu_K(\cancel{M}g\cos\theta)\cancel{R} = \frac{a_{cm}}{\cancel{R}}\left(\frac{1}{2}\cancel{M}R^{\cancel{2}}\right)$

$2\mu_K g\cos\theta = a_{cm}$

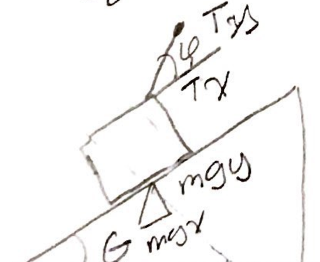
$\cancel{Mg}\sin\theta - \mu_K \cancel{Mg}\cos\theta = 2\cancel{M}\mu_K \cancel{g}\cos\theta$

$+\cancel{\mu_K\cos\theta} \qquad + \mu_K\cos\theta$

$\frac{\sin\theta}{3\cos\theta} = \frac{3\mu_K \cancel{\cos\theta}}{3\cancel{\cos\theta}} \Rightarrow \mu_K = \frac{1}{3}\frac{\sin\theta}{\cos\theta}$

$2\left[\frac{1}{3}\frac{\sin\theta}{\cancel{\cos\theta}}\right] g\cancel{\cos\theta} = a_{cm} = \frac{2}{3}g\cos\theta \; ; \; \boxed{\vec{F} = \frac{2}{3}mg\cos\theta\,\hat{i}}$

$W = \int_0^L \frac{2}{3}mg\cos\theta\,dx = \frac{2mg\cos\theta x}{3}\Big|_0^L = \boxed{\frac{2mg\cos\theta L}{3}}$



WITH PERSON PULLING A BOX

$T_x = T\cos\varphi$

$T_y = T\sin\varphi$

$\Sigma F_y = T_y + N - mg\cos\theta = 0 = T\sin\varphi + N - mg\cos\theta = 0$

$\Sigma F_x = T_x - F_F - mg\sin\theta = 0 = T\cos\varphi - F_F - mg\sin\theta = 0$

$T\sin\varphi + N - mg\cos\theta = 0 \quad -T\sin\varphi + mg\cos\theta$

$-T\sin\varphi \qquad +mg\cos\theta$

$\boxed{N = mg\cos\theta - T\sin\varphi}$ NORMAL

$T\cos\varphi - F_f - mg\sin\theta = 0 \quad -T\cos\varphi + mg\sin\theta$

$-T\cos\varphi \qquad +mg\sin\theta$

$\boxed{F_f = T\cos\varphi - mg\sin\theta}$ FRICTION

$$W_N = \int_0^{\ell} mg\cos\theta - T\sin\varphi \, dx \Rightarrow \boxed{mg\ell\cos\theta - T\ell\sin\varphi}$$

$$W_F = \int_0^{\ell} T\cos\varphi - mg\sin\theta \, dx \Rightarrow \boxed{T\ell\cos\varphi - mg\ell\sin\theta}$$

EXAMPLE

A ROLLER COASTER HAS 8 WHEELS WITH A COMBINED MASS OF 4,500 kg AND TRAVELS 10.00 m ON A SPRING; WHAT IS K FOR THE SPRING AND THE VELOCITY FOR THE ROLLER COASTER? CALCULATE THE TOTAL ENERGY TRAVEL UP THE FIRST HILL ($\theta = 30.00°$) WITH A TENSION OF 2000 N AND A LENGTH OF 25.00 m. WHAT IS THE SPEED OF THE COASTER AT THE BOTTOM OF THE HILL?

$$8\,\frac{2Mg\cos\theta L}{3} = \frac{1}{2}kx^2 \qquad x = L$$

$$2\frac{(16\,Mg\cos\theta\,\cancel{x})}{3\,x} = \frac{2}{2}k\cancel{x^2}\,\frac{x}{x}$$

$$\frac{32Mg\cos\theta}{3x} = k \qquad \theta = 0.000°$$

$$k = \frac{32(4{,}500\,kg)(9.810\,m/s^2)}{3(10.00\,m)}$$

$$k = \boxed{47{,}090\,kg/s^2}$$

$$v = \sqrt{\frac{kx^2}{m}}$$

$$= \sqrt{\frac{(47{,}090\,kg/s^2)(10\,m^2)}{(4{,}500\,kg)}}$$

$$= \boxed{32.35\,m/s}$$

TOTAL ENERGY (E) = $W_N + W_F$ $\qquad \varphi = 0.000°$

$= mgl\cos\theta - Tl\sin\varphi + Tl\cos\varphi - mgl\sin\theta$

$= mgl(\cos\theta - \sin\theta) - Tl(\sin\varphi - \cos\varphi)$

$= mgl(\cos\theta - \sin\theta) + Tl \quad \therefore \sin(0) = 0 \quad \cos(0) = 1$

$= (4{,}500kg)(9.810 m/s^2)(25.00m)(\cos(30.00°) - \sin(30.00°))$

$+ (2000N)(25.00m)$

$= \boxed{454000 J}$

$v = \sqrt{2gh} \qquad h = l\sin\theta$

$v = \sqrt{2gl\sin\theta} = \sqrt{2(9.810 m/s^2)(25.00 m)\sin(30.00°)}$

$\boxed{v = 15.66 m/s}$