4-17-2018 LECTURE NOTES

$\vec{\tau} = \vec{r} \times \vec{F}$ $\quad \vec{F} = m\vec{a}$; $\vec{L} = \vec{r} \times \vec{p}$ $\quad \vec{p} = m\vec{v}$

MAGNITUDE

$\tau = (r \sin\theta) ma$; $a = r\alpha$; $v = r\omega$ $\quad L = (r \sin\theta) mv$

$= r^2 \sin\theta \, m\alpha$; $\lim_{\theta \to 90} \sin\theta = 1$ $\quad = r^2 \sin\theta \, m\omega$

$= mr^2\alpha$; $I = mr^2$ FOR A PARTICLE $\quad = mr^2\omega$

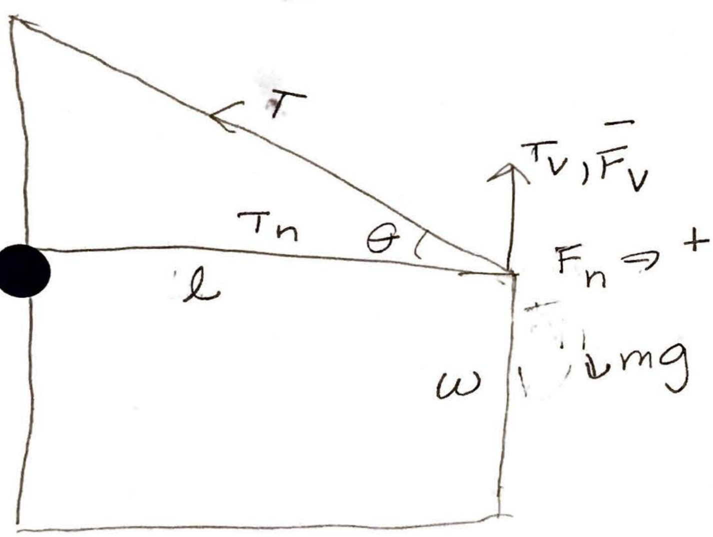
$= I\alpha$; $I = \Sigma m_i r_i^2$ $\quad L = I\omega$

$I = I_{cm} + mh^2$

PARALLEL AXIS THEORUM $\quad \frac{\Delta L}{\Delta t} = I\frac{\omega}{t}$

SIGN

$\frac{\Delta L}{\Delta t} = I\alpha = \tau$



$T = \sqrt{(T_n)^2 + (T_v)^2}$

$\tan\theta = \frac{T_v}{T_h}$

$F_n - T_h = 0 \Rightarrow T_n = F_h$

$F_V + T_V - mg = 0$

$\tau = \vec{r} \times \vec{F} = -F_V(l) + F_h(w) + mg(\frac{1}{2}l) = 0$

$F_V(l) = F_h(w) + mg(\frac{1}{2}l)$

$F_V = F_n(w/l) + \frac{1}{2}mg$

$F_n(w/l) + \frac{1}{2}mg + T_V - mg = 0$

$F_n(w/l) + T_V - \frac{1}{2}mg = 0$

$$\tan\theta = \frac{T_v}{\cancel{T_h}} (\cancel{T_h})$$

$$T_v = T_h \tan\theta \Rightarrow T_v = F_h \tan\theta$$

$$F_h(w/l) + F_h \tan\theta - \cancel{\tfrac{1}{2}mg} = 0$$
$$+\cancel{\tfrac{1}{2}mg} \qquad +\tfrac{1}{2}mg$$

$$F_h(w/l) + F_h \tan\theta = \tfrac{1}{2}mg$$

$$\frac{F_h\,\cancel{[(w/l)+\tan\theta]}}{\cancel{[(w/l)+\tan\theta]}} = \frac{\tfrac{1}{2}mg}{[(w/l)+\tan\theta]}$$

$$\boxed{F_h = \frac{\tfrac{1}{2}mg}{[(w/l)+\tan\theta]}} \qquad \boxed{T_h = \frac{\tfrac{1}{2}mg}{[(w/l)+\tan\theta]}}$$

$$F_v = \frac{\tfrac{1}{2}mg(w/l)}{(w/l)+\tan\theta} + \tfrac{1}{2}mg$$

$$= \frac{\tfrac{1}{2}mg(w/l) + \tfrac{1}{2}mg(w/l) + \tfrac{1}{2}mg\tan\theta}{((w/l)+\tan\theta)}$$

$$\boxed{F_v = \frac{mg(w/l) + \tfrac{1}{2}mg\tan\theta}{((w/l)+\tan\theta)}}$$

$$F_v + T_v - \cancel{mg} = 0$$
$$\cancel{+mg} \qquad +mg$$

$$F_v + T_v = mg$$
$$-F_v \qquad -F_v$$

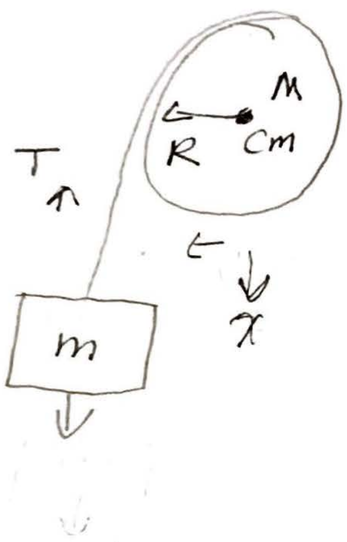
$$T_v = mg - F_v$$

$$= mg - \frac{mg(w/l) + \tfrac{1}{2}mg\tan\theta}{[(w/l)+\tan\theta]}$$

$$T_V = \frac{\cancel{mg(w/2)} + mg\tan\theta - \cancel{mg(w/2)} - \tfrac{1}{2}mg\tan\theta}{[(w/2)+\tan\theta]}$$

$$\boxed{T_V = \frac{\tfrac{1}{2}mg\tan\theta}{[(w/2)+\tan\theta]}}$$

WEIGHTED PULLY



$$L_x = I\omega + mvR$$

$$\tau = mgR = \frac{\Delta L}{\Delta t}$$

$$mgR = \frac{I\omega}{t} + \frac{mRv}{t} \qquad \alpha = a/R$$

$$I = \tfrac{1}{2}MR^2$$

$$a = v/t$$

$$mg\cancel{R} = \tfrac{1}{2}M\cancel{R^2}\left(\frac{a}{\cancel{R}}\right) + m\cancel{R}a$$

$$mg = \tfrac{1}{2}Ma + ma$$

$$\frac{mg}{(\tfrac{1}{2}M+m)} = \frac{a\cancel{(\tfrac{1}{2}M+m)}}{\cancel{(\tfrac{1}{2}M+m)}}$$

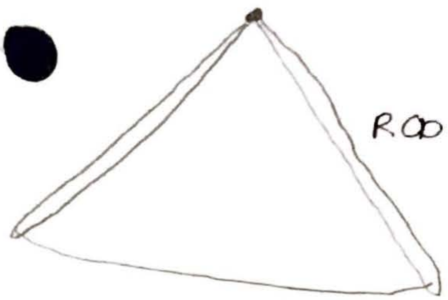
$$a = \frac{2mg}{(M+2m)}$$

$$T_x = F = \frac{Ia}{R^2} = \frac{\tfrac{1}{2}M\cancel{R^2}a}{\cancel{R^2}}$$

$$= \tfrac{1}{\cancel{2}}M\left(\frac{\cancel{2}mg}{(M+2m)}\right)$$

$$\boxed{T_x = \frac{Mmg}{(M+2m)}}$$

PENDULUM



$a = g(1 - \sin\theta)$ $\lim_{\theta \to 0} \sin\theta = 0$

$\tau = I\alpha$

$\frac{\cancel{I}\alpha}{\cancel{I}} = \frac{mgL_{cm}}{I}$

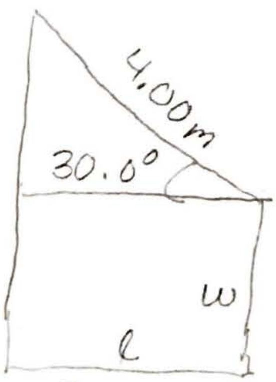
$\alpha = \frac{mgL_{cm}}{I} \Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{mgL_{cm}}{I}$

$\sqrt{\left(\frac{T}{2\pi}\right)^2} = \sqrt{\frac{I}{mgL_{cm}}}$

$\frac{T}{2\pi} = \sqrt{\frac{I}{mgL_{cm}}}$

EXAMPLE

WHAT IS THE TENSION IN A WIRE SUPPORTING A SIGN OF 400 kg? WHAT IS THE MASS OF A PULLEY THAT COULD SUPPORT THE SIGN? IF THE SIGN BREAKS FREE, WHAT IS THE PERIOD OF ITS OSCILLATION?



$T_h = \frac{\frac{1}{2}mg}{\left[(w/l) + \tan\theta\right]}$

$= \frac{\frac{1}{2}(400\,kg)(9.81\,m/s^2)}{\left[(1.25m/3.00m) + \tan(30.0°)\right]} = \boxed{1,970N}$

$T_v = \frac{\frac{1}{2}mg\tan\theta}{\left[(w/l) + \tan\theta\right]}$

$= \frac{\frac{1}{2}(400\,kg)(9.81\,m/s^2)\tan(30.0°)}{\left[(1.25m/3.00m) + \tan(30.0°)\right]} = \boxed{1,140N}$

$$T = \sqrt{T_h^2 + T_v^2} = \sqrt{(1{,}970N)^2 + (1{,}140N)^2} = \boxed{2{,}280\,N}$$

$$(M+2m)$$

$$T = \frac{Mmg}{\cancel{(M+2m)}} \cancel{(M+2m)}$$

$$T(M+2m) = Mmg$$

$$TM + 2Tm = Mmg$$

$$-TM \qquad -TM$$

$$2Tm = Mmg - TM$$

$$\frac{2Tm}{(mg-T)} = \frac{M\cancel{(mg-T)}}{\cancel{(mg-T)}}$$

$$M = \frac{2Tm}{(mg-T)} = \frac{2(2{,}280\,N)(400\,kg)}{((400\,kg)(9.81\,m/s^2) - (2{,}280\,N))} = \boxed{187\,kg}$$

1,110 kg

$$T = 2\pi\sqrt{\frac{I}{mgL_{cm}}} = 2\pi\sqrt{\frac{\frac{1}{12}\cancel{m}(L^2+w^2)}{\cancel{m}gL_{cm}}}$$

$$= 2\pi\sqrt{\frac{\frac{1}{12}((1.25m)^2 + (3.00m)^2}{(9.81\,m/s^2)(4.00\,m)}}$$

$$= \boxed{0.945}$$