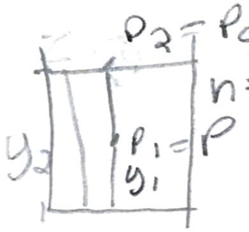
4-14-2021 LECTURE NOTES

PRESSURE AND BERNOULLI'S EQUATION OR FLUID MECHANICS

$P = \frac{\Delta F}{\Delta A} = \frac{\Delta F}{\Delta x \Delta y}$; $\rho = \frac{\Delta m}{\Delta V} = \frac{m}{V}$ UNIT = 1 PASCAL = $\frac{N}{m^2}$ = 1 Pa

PRESSURE DENSITY

STATIC FLUID IN A CONTAINED VOLUME



$h = y_2 - y_1$

$\Sigma F_y = \cancel{pA} - (\cancel{p} + \Delta P)A - \rho g A \Delta y = 0$

$-\Delta P A$

$-\Delta P \cancel{A}$

$\frac{+\Delta P \cancel{A}}{+\Delta y} = \frac{+\rho g \Delta y \cancel{A}}{+\Delta y}$

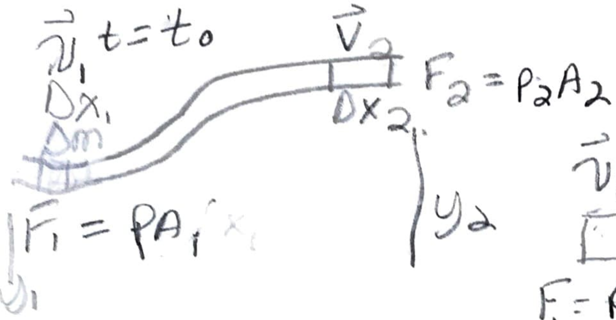
$\frac{\Delta P}{\Delta y} = \rho g$ CONVERTS TO THIS INTEGRATIO

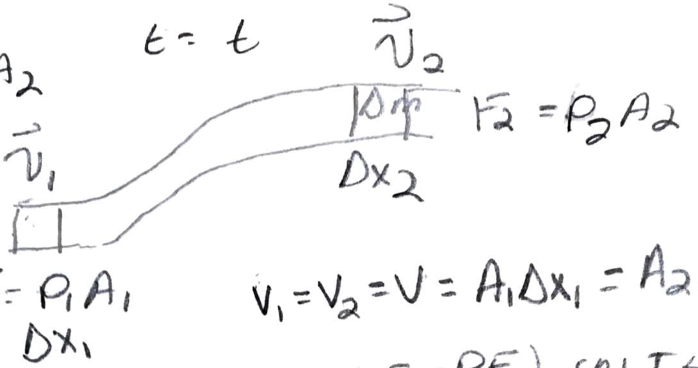
$\int_{P}^{P_0} dP = -\int_{y_0}^{y} \rho g \, dy$

$P_0 - P = -\rho g (y - y_0)$

$\boxed{P = P_0 + \rho g h}$

MOVING FLUID IN A CONTAINED VOLUME





$V_1 = V_2 = V = A_1 \Delta x_1 = A_2 \Delta x_2$

WE USE THE <u>LAGRAGIAN</u> ($L = KE - PE$) IN THIS CASE. $W = \vec{F} \cdot \vec{d}$; $V = A\Delta x = \Delta m/\rho$ $\quad L = 0$ SINCE CONSERVED

$$W_{EXT} = W_1 + W_2 + W_3 = PE$$
$$= P_1 A_1 \Delta x_1 + (-P_2 A_2 \Delta x_2) + [\Delta m g (y_2 - y_1)]$$
$$= (P_1 - P_2)(\Delta m/\rho) - \Delta m g (y_2 - y_1)$$

$$KE = \tfrac{1}{2} \Delta m v_2^2 - \tfrac{1}{2} \Delta m v_1^2$$

$$L = KE - PE \Rightarrow KE = PE$$

$$\tfrac{1}{2} \cancel{\Delta m} v_2^2 - \tfrac{1}{2} \cancel{\Delta m} v_1^2 = (P_1 - P_2)(\cancel{\Delta m}/\rho) - \cancel{\Delta m} g (y_2 - y_1)$$

$$\rho [\tfrac{1}{2} v_2^2 + P_2/\rho + g y_2] = \rho [\tfrac{1}{2} v_1^2 + P_1/\rho + g y_1]$$

$$P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 = P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y$$ SINCE AT ARBITRARY POINTS

$$\therefore P + \tfrac{1}{2}\rho v^2 + \rho g y_i = CONST$$

WHAT IS THE VELOCITY OF THE SPACEX STARSHIP (11,7900 kg) FUEL WHEN THE SPACE CRAFT IS MOVING? AND THE CHANGE IN PRESSURE?

$$\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 = \frac{(P_1 - P_2)}{\rho} \cancel{- g(y - y_0)}$$

IN A ROCKET WE CAN NEGLECT Dy AND ON

$$2\,\frac{1}{2}(v_0^2 - v^2) = \frac{2(P_0 - P)}{\rho}$$

$$v_0^2 - v^2 = \frac{2(P_0 - P)}{\rho} + v^2$$

$+v$

$$v_0^2 = \frac{-2(P_0 - P)}{\rho} + v^2$$

$$v_0^2 = \frac{-2(P_0 - P)}{\rho}$$

THRUST => $v_0 \frac{\Delta m}{\Delta t}$

THRUST / $\Delta m / \Delta t = v_0 = 1.2 \times 10^7 N$

SPECIFIC IMPULSE = 380s

$\Delta m = 1.32 \times 10^6$ KG

$\rho = 1.14 g/cm^3 \left(\frac{1 kg}{1000 g}\right)\left(\frac{100 cm}{1 m}\right)^3$

$= 1,140 \ kg/m^3$

$$\frac{1.200 \times 10^7 N}{\frac{(1.32 \times 10^6 kg)}{380s}} = v_0$$

$$\boxed{3,450 m/s = v_0}$$

$$v_0^2 = \frac{2 \Delta P}{\rho}$$

$$\frac{v_0^2 \rho}{2} = \Delta P = \frac{(3,450 m/s)^2 (1,140 kg/m^3)}{2} = \Delta P = \boxed{6.78 \times 10^9 Pa}$$