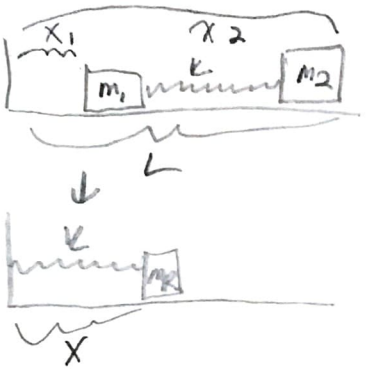
4-12-2021 LECTURE NOTES

COUPLED HARMONIC OSCILLATOR



$x = (x_1 - x_2) - L$

$m_R = \frac{m_1 m_2}{(m_1 + m_2)}$

IN THE CASE OF A SPRING WE USE THE HAMILTONIAN $H = KE + PE$. HAMILTONIANS CAN BE ADDED, I.E. $H_{TOT} = H_1 + H_2 + H_3 + \ldots + H_N$. WE ADD SPRING 1 TO SPRING 2.

$H_{TOT} = H_{S1} + H_{S2}$ AND $H_{TOT} = 0$ FOR CONSERVATION OF ENERGY

$0 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} k x^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} k x^2$; $v_1 = v_2 = v$

$= v^2(m_1 + m_2) + 2kx^2$

$-v^2(m_1 + m_2) = -v^2(m_1 + m_2)$

$+\frac{v^2(m_1 + m_2)}{+(m_1 + m_2)} = \frac{2kx^2}{-(m_1 + m_2)}$

$v^2 = -\frac{2kx^2}{(m_1 + m_2)}$

$v^2 - v_0^2 = 2da$

$d = x$; $v_0 = 0.00 m/s$

$v^2 = 2xa$

$$\Sigma F_x = a = -\frac{2kx^2}{(m_1+m_2)}$$

$$a = -\frac{kx}{(m_1+m_2)} \cdot \frac{(m_1+m_2)^2}{m_1 m_2}$$

$$= -\frac{kx(m_1+m_2)}{m_1 m_2}$$

$$\boxed{a = -\frac{kx}{m_R}}$$

A SOLUTION OF HCL (HYDROCHLORIC ACID) CONSISTS OF 76% ^{35}Cl AND 24% ^{37}Cl AND 100% H ($m_{^{35}Cl} = 34.968u$; $m_{^{37}Cl} = 36.965u$; $m_H = 1.0078u$). WHAT IS ITS REDUCED MASS? WHAT IS THE FORCE CONSTANT (k) IF THE FREQUENCY IS 8.5000×10^{13} Hz ($f = \frac{\omega}{2\pi}$)? $u = 1.66000 \times 10^{-27}$ kg

$$m_R = \frac{m_1 m_2}{(m_1+m_2)}; \quad \mu_{^{35}Cl} = \frac{(34.968u)(1.0078u)}{(34.968u+1.0078u)} = 0.9796u$$

$$\mu_{^{37}Cl} = \frac{(36.965u)(1.0078u)}{(36.968u+1.0078u)} = 0.9810u$$

WE KNOW $\omega = \sqrt{\frac{k}{m}} \Rightarrow \omega_R = \sqrt{\frac{k}{m_R}} \Rightarrow f_k = \frac{1}{2\pi}\sqrt{\frac{k}{m_R}}$

$m_R = (0.76)\,{}^{1}H^{35}Cl + (0.24)\,{}^{1}H^{37}Cl$

$= (0.76)(0.97910u) + (0.24)(0.9810u)$

$= \boxed{0.9799u}$

$(f\,2\pi)^2 = \left(\sqrt{\frac{k}{m_R}}\right)^2$

$4\pi^2 f^2 = \frac{k}{m_R}$

$k = 4\pi^2 f^2 m_R = 4\pi^2 (8.5000\times10^{13}\,Hz)^2 (0.9799)(1.6600\times10^{-27}\,kg)$

$= \boxed{463.97\,kg/s^2}$