4/10/2018 LECTURE NOTES

COLLISIONS

ELASTIC — P AND KE CONSERVED

INELASTIC — P CONSERVED,

$\xrightarrow{v_1} \bullet m_1 \quad \bullet m_2 \Rightarrow \xrightarrow{v_1'} \bullet m_2 \quad \xrightarrow{v_2'} \bullet m_2$

ELASTIC ; $v_2 = 0.00\,m/s$

$P = P'$; $P = \sum m_i v_i$

$KE = KE'$; $KE = \sum \frac{1}{2} m_i v_i^2$

$$\frac{m_1 v_1}{m_1} = \frac{m_1 v_1' + m_2 v_2'}{m_1} \; ; \; \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$v_1 = \frac{m_1 v_1' + m_2 v_2'}{m_1}$$

$$\frac{1}{2} m_1 \left(\frac{m_1 v_1' + m_2 v_2'}{m_1}\right)^2 m_1 = \left(\frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2\right) m_1$$

$$m_1^2 v_1'^2 + 2 m_1 m_2 v_1' v_2' + m_2^2 v_2'^2 = m_1^2 v_1'^2 + m_1 m_2 v_2'^2$$

$$-m_1^2 v_1'^2 \qquad\qquad -m_1^2 v_1'^2$$

$$2 m_1 m_2 v_1' v_2' + m_2^2 v_2'^2 = m_1 m_2 v_2'^2 - m_2^2 v_2'^2$$

$$-m_2^2 v_2'^2$$

$$\frac{2 m_1 m_2 v_1' v_2'}{2 m_1 m_2 v_2'} = \frac{m_1 m_2 v_2'^2}{2 m_1 m_2 v_2'} - \frac{m_2^2 v_2'^2}{2 m_1 m_2 v_2'}$$

$$v_1'^2 = \frac{1}{2}\left(\frac{m_1 - m_2}{m_1}\right) v_2'$$

$$v_1 = \frac{\cancel{m_1}\left[\frac{1}{2}\left(\frac{m_1-m_2}{\cancel{m_1}}\right)\right]v_2' + m_2 v_2'}{m_1}$$

$$v_1 m_1 = \frac{\frac{1}{2}m_1 v_2' - \frac{1}{2}m_2 v_2' + m_2 v_2' \cancel{m_1}}{\cancel{m_1}}$$

$$m_1 v_1 = \tfrac{1}{2}m_1 v_2' - \tfrac{1}{2}m_2 v_2' + m_2 v_2'$$

$$= \left(\tfrac{1}{2}m_1 v_2' + \tfrac{1}{2}m_2 v_2'\right)2$$

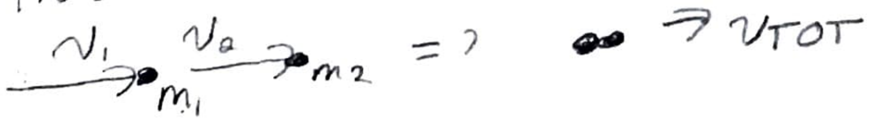
$$\frac{2m_1 v_1}{(m_1+m_2)} = \frac{\cancel{(m_1+m_2)}\,v_2'}{\cancel{(m_1+m_2)}}$$

$$\boxed{v_2' = \frac{2m_1 v_1}{(m_1+m_2)}}$$

$$v_1' = \frac{1}{\cancel{2}}\left(\frac{m_1-m_2}{\cancel{m_1}}\right)\left(\frac{\cancel{2m_1}v_1}{m_1+m_2}\right)$$

$$\boxed{v_1' = \left(\frac{m_1-m_2}{m_1+m_2}\right)v_1}$$

INELASTIC $p = p'$ ENERGY RATIO $\frac{KE'}{KE}$



$$\frac{m_1 v_1 + m_2 v_2}{m_{TOT}} = \frac{\cancel{m_{TOT}}\, v_{TOT}}{\cancel{m_{TOT}}}$$

$$v_{TOT} = \frac{m_1 v_1 + m_2 v_2}{m_{TOT}}; \quad \frac{KE'}{KE} = \frac{\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m v_2^2}{\frac{1}{2}m_{TOT} v_{TOT}^2}$$

EXAMPLE

TWO BILLARD BALLS ($m_1 = 0.34$ kg ; $m_2 = 0{,}40$ kg) ARE ON A POOL TABLE. WHAT ARE THEIR FINAL VELOCITIES IF THE FIRST IS GOING AT 1,20 m/s ?

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 = \left(\frac{0.34\,kg - 0.10\,kg}{0.34\,kg + 0.10\,kg}\right)(1.20\,m/s)$$

$$= \boxed{0.66\,m/s}$$

$$v_2' = \frac{2m_1}{(m_1 + m_2)} v_1 = \frac{2(0.34\,kg)(1.20\,m/s)}{(0.34\,kg + 0.10\,kg)} = \boxed{1.85\,m/s}$$

EXAMPLE

TWO PLANES ($m_1 = m_2 = 715{,}000$ kg) CRASH AT THE AIRPORT AT TENRIFE. WHAT IS THEIR FINAL VELOCITY IF THEY ARE GOING AT $v_1 = 375$ m/s AND $v_2 = 350$ m/s?

$$v_{TOT} = \frac{m_1 v_1 + m_2 v_2}{m_{TOT}} \qquad m_1 = m_2$$

$$= \frac{m_1 (v_1 + v_2)}{2 m_1}$$

$$= \frac{(375\,m/s + 350\,m/s)}{2} = \boxed{363\,m/s}$$