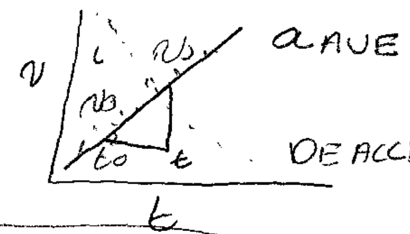
2/6/2018 LECTURE NOTES PHY 2053

GALILEO NOTED THAT VELOCITIES DO CHANGE WITH RESPECT TO TIME WITH HIS EXPERIMENTS FEATHERS AND SPHERES FALLING DOWN FROM TOWERS AND INCLINE PLANES. OBSERVING THE MOTION HE MODELED A NEW CONCEPT, ACCELERATION.

ACCELERATION IS A RATIO OF VELOCITY TO TIME:



ACCELERATION

$$a = \frac{v - v_0}{t - t_0}$$

DE ACCELERATION

OR IN VECTORS

$$\vec{a} = \frac{\vec{v} - \vec{v}_0}{t - t_0} \qquad t - t_0 = t_d$$

$$\vec{a} = \frac{\vec{v} - \vec{v}_0}{t_d}$$

$$\boxed{\vec{a}_{AVE} = \frac{\vec{v} - \vec{v}_0}{t_d}}$$

AND FOR $\vec{v} - \vec{v}_0 \Rightarrow \vec{v}$

AND $t_d \Rightarrow t$

$$\boxed{\vec{a} = \frac{\vec{v}}{t}}$$

LINEAR ACCELERATION

NOW WE KNOW THE MAGNITUDES OF VELOCITY:

$$\bar{v} = \frac{r - r_0}{t} \quad ; \quad \bar{v} = \frac{v + v_0}{2}$$

AND THE MAGNITUDE OF ACCELERATION: $a = \frac{v - v_0}{t}$

NOW WE COMBINE THE THREE EQUATION AND SOLVE FOR s

$$\bar{v} = \bar{v} \Rightarrow \frac{r - r_0}{t} = \frac{v + v_0}{2} \quad ; \text{ NOW SOLVE FOR } v$$

$$s = r - r_0$$

$$\frac{2s}{t} = \frac{v+v_0}{\cancel{2}}\cancel{2}$$

$$\frac{2s}{t} = v + \cancel{v_0}$$

$$-v_0 \qquad\qquad -\cancel{v_0}$$

$$\boxed{\frac{2s}{t} - v_0 = v}$$ NOW SUBSITUTE INTO a

$$a = \frac{v}{t} - \frac{v_0}{t}$$

$$at = \left[\frac{1}{\cancel{t}}\left(\frac{2s}{t} - v_0\right] - \frac{v_0}{\cancel{t}}\right]\cancel{t}$$

$$at = \left(\frac{2s}{t} - v_0\right] - v_0$$

$$at^2 = \cancel{t}\frac{2s}{\cancel{t}} - 2v_0 t$$

$$\frac{at^2}{2} = \frac{\cancel{2}s}{\cancel{2}} - \frac{\cancel{2}v_0 t}{\cancel{2}}$$

$$v_0 t + \frac{at^2}{2} = s - v_0 t + v_0 t$$

$$\boxed{\frac{at^2}{2} + v_0 t = s}$$

THIS IS USEFUL IF ONE KNOWS VELOCITY AND TIME; BUT WE CAN SUBSITUTE $t = \frac{v - v_0}{a}$ INTO t SO WE CAN USE DISPLACEMENT AND VELOCITY.

$$\frac{a}{2}\left(\frac{v-v_0}{a}\right)^2 + v_0\left(\frac{v-v_0}{a}\right) = s$$

$$\frac{1}{2a}(v^2 - 2vv_0 + v_0^2) + \frac{1}{a}(vv_0 - v_0^2) = s \qquad 2a \qquad 2a$$

$$v^2 - \cancel{2vv_0} + \cancel{v_0^2} + \cancel{2vv_0} - 2\cancel{v_0^2} = 2as$$

$$\boxed{v^2 - v_0^2 = 2as}$$

EXAMPLE

NOW TAKE THE DISPLACEMENT POSITIONS FROM THE 1/23 LECTURE AND FIND THE INSTANTANEOUS VELOCITIES IF $t = 3.23\ s$ AND $t_0 = 1.54\ s$. FROM THIS FIND THE ACCELERATION. IF THE OBJECT TRAVELS 25.0 m FURTHER AT THIS ACCELERATION, HOW LONG DOES TAKE AND WHAT IS THE FINAL VELOCITY?

$\vec{r}_1 = \vec{r}_0$ AND $\vec{r}_2 = \vec{r}$

$$\vec{v} = \frac{\vec{r}}{t} = \frac{85.0\,m}{3.23\,s}\hat{i} + \frac{85.0\,m}{3.25\,s}\hat{j} = 26.2\,m/s\,\hat{i} + 26.2\,m/s\,\hat{j}$$

$$|\vec{v}| = v = \sqrt{(26.2\,m/s)^2 + (26.2\,m/s)^2} = 37.1\,m/s$$

$$\vec{v}_0 = \frac{\vec{r}_0}{t_0} = \frac{22.5\,m}{1.54\,s}\hat{i} + \frac{39.0\,m}{1.54\,s}\hat{j} = 14.6\,m/s\,\hat{i} + 25.3\,m/s\,\hat{j}$$

$$|\vec{v}_0| = v_0 = \sqrt{(14.6\,m/s)^2 + (25.3\,m/s)^2} = 29.2\,m/s$$

$$a = \frac{v - v_0}{t - t_0} = \frac{37.1\,m/s - 29.2\,m/s}{3.23\,s - 1.54\,s} = 4.67\,m/s^2$$

NOW TO FIND FINAL VELOCITY THE FINAL v BECOMES v_0

$v^2 - v_0^2 = 2sa$ AND SOLVE FOR v

$$v^2 - \cancel{v_0^2} = 2sa$$
$$+\cancel{v_0^2} \qquad +v_0^2$$

$s = 25.0\,m$
$a = 4.67\,m/s^2$
$v_0 = 37.1\,m/s$

$$v^2 = 2sa + v_0^2$$

$$\sqrt{v^2} = \sqrt{2sa + v_0^2}$$

$$v = \sqrt{2sa + v_0^2} = \sqrt{2(25.0m)(4.67m/s^2) + (37.1m/s)^2}$$
$$= \boxed{40.1\,m/s} \text{ AFTER } 25.0m$$

TO FIND THE TIME USE

$$t\,\frac{2s}{t}\overset{t}{-}v_0 \overset{t}{=} v$$

$s = 25.0m$
$v_0 = 37.1\,m/s$
$v = 40.1\,m/s$

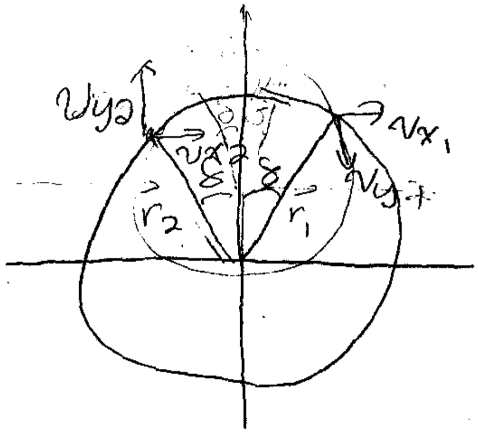
$$2s - \cancel{v_0 t} = vt$$
$$+\cancel{v_0 t} \quad + v_0 t$$

$$2s = v_0 t + vt$$

$$\frac{2s}{(v_0+v)} = \frac{\cancel{(v_0+v)}t}{\cancel{(v_0+v)}}$$

$$t = \frac{2s}{(v+v_0)} = \frac{2(25.0m)}{(40.1m/s + 37.1m/s)} = \boxed{0.65\,s}$$

ANGULAR ACCELERATION



$$\vec{v}_2 = v_{x2}\hat{i} - v_{y2}\hat{j} \qquad v_x = v\cos\delta$$
$$\vec{v}_1 = v_{x1}\hat{i} + v_{y1}\hat{j} \qquad v_y = v\sin\delta$$
$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t} \qquad |\vec{a}| = a = \frac{v_2 - v_1}{t}$$

$$\vec{v}_2 - \vec{v}_1 = (v_{x2} - v_{x1})\hat{i} + (-v_{y2} - v_{y1})\hat{j} \qquad v = v_y = v_x$$
$$= 0\hat{i} - 2v_y\hat{j}$$

$$a = \frac{-2v_y}{t} = \frac{-2v\sin\delta}{t}$$

$$\leftarrow v = \frac{l}{t} = \frac{2r\delta t}{t} \Rightarrow \frac{\cancel{v}t}{\cancel{v}} = \frac{2r\delta}{v} \Rightarrow t = \frac{2r\delta}{v}$$

$$a = \frac{-\cancel{2}v\sin\delta}{\frac{\cancel{2}r\delta}{v}}$$

$$a_{AVERAGE} = -\frac{v^2}{r}\left(\frac{\sin\delta}{\delta}\right) \qquad \lim_{\delta\to 0}\left(\frac{\sin\delta}{\delta}\right) = 1$$

$$a_{INSTANTANEOUS} = \frac{v^2}{r}$$

EXAMPLE
USING THE ANGULAR DATA FROM THE 1/23 LECTURE TO FIND THE AVERAGE AND INSTANTANEOUS ACCELERATION

$v = 6.23\,m/s$; $\delta = \pi/6$; $r = 25.0\,m$

$$a_{AVE} = -\frac{(6.23\,m/s)}{25.0\,m}\left(\frac{\sin\pi/6}{\pi/6}\right) = \boxed{-1.48\,m/s^2}$$

$$a_{INST} = \frac{(6.23\,m/s)^2}{25.0\,m} = \boxed{1.55\,m/s}$$