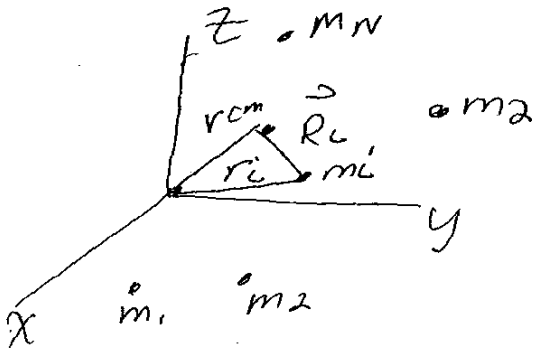
2/20/2018 LECTURE NOTES

CENTER OF MASS (CM)



$\sum m_i \vec{r} = r_{cm} \sum m_i + \sum m_i R_i$

$-\sum m_i R_i \qquad -\sum m_i R_i$

$r_{cm} \sum m_i = \sum m_i \vec{r} - \sum m_i R_i$ $\lim_{R_i \to 0} \sum m_i R_i \neq 0$ AT CENTER OF MASS

$\vec{r}_{cm} m_{TOT} = \sum m_i \vec{r}_i$ $\therefore \sum m_i = m_{TOT}$

$\frac{\vec{r}_{cm} m_{TOT}}{m_{TOT}} = \frac{\sum m_i \vec{r}_i}{m_{TOT}}$

$$\vec{r}_{cm} = \boxed{\frac{\sum m_i \vec{r}_i}{m_{TOT}}}$$

$$\vec{v}_{cm} = \boxed{\frac{D\vec{r}_{cm}}{Dt}} = \frac{D}{Dt}\left[\frac{\sum m_i \vec{r}_i}{t}\right] = \frac{1}{m_{TOT}} \sum m_i \frac{D\vec{r}_i}{Dt}$$

$$= \boxed{\frac{1}{m_{TOT}} \sum m_i \vec{v}_i}$$

$$\vec{P}_{cm} = \sum m_i v_i = m_{TOT} \vec{v}_{cm}$$

$KE = \frac{1}{2} m v^2$

$KE_{sys} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_2^2 + \ldots + \frac{1}{2} m_N v_N^2$

$\vec{r}_i = \vec{r}_{cm} + \vec{R}_i \Rightarrow \vec{r}_i - \vec{r}_{cm} = \vec{R}_i$

$\frac{D\vec{r}}{Dt} = \frac{D\vec{r}_{cm}}{Dt} + \frac{D\vec{R}_i}{Dt}$

$\vec{v}_i = \vec{v}_{cm} + \vec{u}_i =$

$\vec{v}_i \cdot \vec{v}_i = (\vec{v}_{cm} + \vec{u}_i)(\vec{v}_{cm} + \vec{u}_i)$

$= (v_{cm}^2 + 2\vec{v}_{cm} \cdot \vec{u}_i + \vec{u}_i)$ ($\vec{u}_i = 0$)

$KE_{sys} = \frac{1}{2} m (v_{cm}^2 + u_i^2)$

$KE_{sys} = \frac{1}{2} m_{TOT} v_{cm}^2 + \frac{1}{2} \Sigma m_i u_i^2$

KE_{cm} $\quad$ KE_{INT}

EXAMPLE

USING THESE TWO VECTORS

$\vec{r}_1 = 45.0\,m\,\hat{i} + 26.0\,m\,\hat{j}$ $\quad (m_1 = 20.4\,kg)$

$\vec{r}_2 = 79.7\,m\,\hat{i} + 79.7\,m\,\hat{j} + 41.0\,m\,\hat{k}$ $\quad (m_2 = 3.21\,kg)$

$t = 39.1\,s$

WHAT IS $\vec{r}_{cm}$; $\vec{v}_{cm}$; $\vec{p}_{cm}$; KE_{cm}; KE_{INT} ?

$m_{TOT} = 20.0\,kg + 3.21\,kg = 23.2\,kg$

$m_1 \vec{r}_1 = (20.0\,kg)(45.0\,m\,\hat{i} + 26.0\,m\,\hat{j})$

$+ \quad = 900\,kg\,m\,\hat{i} + 5{,}200\,kg\,m\,\hat{j}$

$m_2 \vec{r}_2 = (3.21\,kg)(79.7\,m\,\hat{i} + 79.7\,m\,\hat{j} + 41.0\,m\,\hat{k})$

$= 255\,kg\,m\,\hat{i} + 255\,kg\,m\,\hat{j} + 132\,kg\,m\,\hat{k}$

$\vec{r}_{cm} = \frac{1{,}160\,kg\,m\,\hat{i} + 5{,}460\,kg\,m\,\hat{j} + 132\,kg\,m\,\hat{k}}{23.2\,kg}$

$$\vec{r}_{cm} = 50.0\,m\,\hat{i} + 23.5\,m\,\hat{j} + 5.69\,m\,\hat{k}$$

$$\vec{v}_{cm} = \frac{\vec{r}_{cm}}{t} = \frac{50.0\,m\,\hat{i} + 23.5\,m\,\hat{j} + 5.69\,m\,\hat{k}}{39.1\,s}$$

$$= 1.28\,m/s\,\hat{i} + 6.01\,m/s\,\hat{j} + 0.15\,m/s\,\hat{k}$$

$$\vec{p}_{cm} = m_{TOT}\vec{v}_{cm} = (23.2\,kg)(1.28\,m/s\,\hat{i} + 6.01\,m/s\,\hat{j} + 0.15\,m/s\,\hat{k})$$

$$= 29.7\,kg\,m/s\,\hat{i} + 139\,kg\,m/s\,\hat{j} + 3.48\,kg\,m/s\,\hat{k}$$

$$KE_{cm} = \tfrac{1}{2} m_{TOT} v_{cm}^2 \quad \therefore v_{cm}^2 = \vec{v}_{cm} \cdot \vec{v}_{cm}$$

$$= \tfrac{1}{2}(23.2\,kg)((1.28\,m/s)^2 + (6.02\,m/s)^2 + (0.15\,m/s)^2)$$

$$= 441\,J$$

$$\vec{r}_1 - \vec{r}_{cm} = (45.0\,m - 50.0\,m)\hat{i} + (260\,m - 235\,m)\hat{j} + (0 - 5.69\,m)\hat{k}$$

$$= -5.00\,m\,\hat{i} + 25.0\,m\,\hat{j} - 5.69\,m\,\hat{k} = \vec{R}_1$$

$$\vec{u}_1 = \frac{\vec{R}_1}{t} = -0.13\,m/s\,\hat{i} + 0.64\,m/s\,\hat{j} - 0.14\,m/s\,\hat{k}$$

$$KE_{INT1} = \tfrac{1}{2} m_1 u_1^2 =$$

$$= \tfrac{1}{2}(20.0\,kg)((-0.13\,m/s)^2 + (0.64\,m/s)^2 + (-0.14\,m/s)^2)$$

$$= 4.46\,J$$

$$\vec{r}_2 - \vec{r}_{cm} = (79.7\,m - 50.0\,m)\hat{i} + (79.7\,m - 235\,m)\hat{j} + (41.9\,m - 5.69\,m)\hat{k}$$

$$= -29.7\,m\,\hat{i} - 155\,m\,\hat{j} + 35.3\,m\,\hat{k} = R_2$$

$$\vec{u}_2 = \frac{\vec{R}_2}{t} = -0.76\,m/s\,\hat{i} - 3.96\,m/s\,\hat{j} + 0.90\,m/s\,\hat{k}$$

$$KE_{INT2} = \frac{1}{2} m_2 v_2^2$$

$$= \frac{1}{2}(3.21\,kg)((-0.760\,m/s)^2 + (-3.96\,m/s)^2 + (0.90\,m/s)^2)$$

$$= 27.4\,J$$

$$KE_{INT} = KE_{INT1} + KE_{INT2}$$

$$= 4.46\,J + 27.4\,J$$

$$= \boxed{31.9\,J}$$