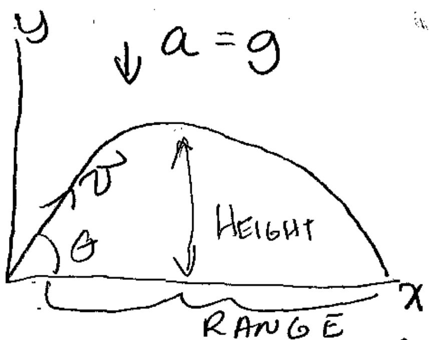
2/13/2018 LECTURE NOTES

PHY 2053

NOW WE ARE GOING TO SEE HOW AN OBJECT MOVES IN TWO DIMENSION ON EARTH. FROM OUR LAST LECTURE (2/6/2018) WE HAVE THESE EQUATIONS:

$$a = \frac{v - v_0}{t}\ ;\quad \frac{v + v_0}{2} = \frac{r - r_0}{t}\ ;\quad s = \frac{at^2}{2} + v_0 t$$

NOW IN OUR GRAVITY FIELD WE SEE AN ARC:



$v_{0x} = v_0 \cos\theta$

$v_{0y} = v_0 \sin\theta$

THIS DEMONSTRATION SHOWS WHAT GOES UP, MUST GOES DOWN.

$a = a_x \hat{i} + a_y \hat{j}$; $a_x = 0\,m/s$; $a_y = -g$ (ACCELERATION DUE TO GRAVITY) $9.81\,m/s^2$

$$\boxed{a_x = 0\,m/s^2}$$

$$a_x = \frac{v_x - v_{0x}}{t}$$

$$t\,0\,m/s^2 = \frac{v_x - v_{0x}}{t}\,t$$

$$0 = v_x - v_{0x}$$
$$+v_{0x} \qquad +v_{0x}$$

$$\boxed{v_x = v_{0x}}$$

$$\frac{v + v_0}{2} = \frac{r - r_0}{t}$$

$$\frac{v_{0x} + v_{0x}}{2} = \frac{x - x_0}{t}$$

$$\boxed{a_y = -g}$$

$$a_y = \frac{v_y - v_{y0}}{t}$$

$$-g\,t = \frac{v_y - v_{y0}}{t}\,t$$

$$-gt = v_y - v_{y0}$$
$$+v_{y0} \qquad +v_{y0}$$

$$\boxed{v_y = v_{y0} - gt}$$

$$\frac{v + v_0}{2} = \frac{r - r_0}{t}$$

$$\frac{v_{y0} - gt + v_0}{2} = \frac{y - y_0}{t}$$

$$t\, v_{0x} = \frac{x - x_0}{\cancel{t}}\cancel{t}$$

$$v_{0x} t = x - x_0$$
$$+x_0 \quad +x_0$$

$$\boxed{x = v_{0x} t + x_0}$$

$$\cancel{t}\left(\frac{v_{y0} - gt + v_{y0}}{2}\right) = \left(\frac{y - y_0}{\cancel{t}}\right)t$$

$$v_{0y} t - \frac{gt^2}{2} = y - y_0$$
$$+y_0 \quad +y_0$$

$$\boxed{-\frac{gt^2}{2} + v_{0y} t + y_0 = y}$$

NOW WE WANT TO FIND HEIGHT ($v_y = 0\,m/s$)

$$v_y = v_{y0} - gt$$
$$-v_{y0} \quad 0 = v_{y0} - gt - v_{y0}$$
$$+v_{y0} = +gt$$
$$\frac{v_0 \sin\theta}{g} = \frac{gt}{g}$$
$$\boxed{\frac{v_0 \sin\theta}{g} = t_{HEIGHT}}$$

$$-\frac{gt^2}{2} + v_{0y} t + y_0 = y$$
$$-\frac{g}{2}\left(\frac{v_0 \sin\theta}{g}\right)^2 + v_0 \sin\theta\left(\frac{v_0 \sin\theta}{g}\right) + y_0 = y$$
$$-\frac{v_0^2 \sin^2\theta}{2g} + \frac{v_0^2 \sin^2\theta}{g} + y_0 = y$$
$$\frac{-v_0^2 \sin^2\theta + 2v_0^2 \sin\theta}{2g} + y_0 = y$$
$$\boxed{\frac{v_0^2 \sin^2\theta}{2g} + y_0 = y} \quad \text{HEIGHT}$$

NOW WE WANT TO FIND RANGE ($v_x = 0\,m/s;\ y = 0$)

FOR TIME

$$\underbrace{-\frac{g}{2}}_{a} t^2 + \underbrace{v_{0y}}_{b} t + \underbrace{y_0}_{c} = 0$$

NOW WE USE THE QUADRATIC FORMULA

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(v_0 \sin\theta) \pm \sqrt{(v_0^2 \sin^2\theta) - \cancel{4}(-g/\cancel{2})(y_0)}}{2(-g/2)}$$

ONLY + IS THE PHYSICALLY RIGHT SOLUTION

$$\boxed{t_{RANGE} = \frac{v_0 \sin\theta}{g} + \sqrt{\left(\frac{v_0 \sin\theta}{g}\right)^2 + \frac{2y_0}{g}}}$$

NOW PLUG IN t_{RANGE} INTO $x = v_{0x} t + x_0$

$$\boxed{x = v_0 \cos\theta \left[\frac{v_0 \sin\theta}{g} + \sqrt{\left(\frac{v_0 \sin\theta}{g}\right)^2 + \frac{2y_0}{g}} \right] + x_0}$$ RANGE

AND THE FINAL VELOCITY

$$\boxed{v_f = v_0 \cos\theta \, \hat{i} + (v_0 \sin\theta - g t_{RANGE}) \hat{j}}$$

EXAMPLE

WHAT IS THE RANGE AND HEIGHT OF A PROJECTILE STARTING AT $x = 0.00\,m$; $y = 0.00\,m$, WITH A VELOCITY OF $2.51\,m/s$ AND $\theta = 35.0°$? FINAL VELOCITY

$$HEIGHT = \frac{v^2 \sin^2\theta}{2g} + y_0$$

$$= \frac{(2.51\,m/s)^2 \sin^2(35.0°)}{2(9.81\,m/s^2)} + 0.00\,m$$

$$HEIGHT = \boxed{0.11\,m}$$

$$RANGE = v_0 \cos\theta \left[\frac{v_0 \sin\theta}{g} + \sqrt{\left(\frac{v_0 \sin\theta}{g}\right)^2 + \frac{2\cancel{y_0}^{\,0}}{g}} \right] + \cancel{x_0}^{\,0}$$

$$= v_0 \cos \left[\frac{v_0 \sin\theta}{g} + \frac{v_0 \sin\theta}{g} \right]$$

$$= \frac{2 v_0^2 \cos\theta \sin\theta}{g}$$

$$= \frac{2(2.51\,m/s)^2 \cos(35°) \sin(35°)}{9.81\,m/s^2}$$

$$= \boxed{0.61\,m}$$

$$t_{RANGE} = \frac{v_0 \sin\theta}{g} + \sqrt{\left(\frac{v_0 \sin\theta}{g}\right)^2 + \cancel{\frac{2y_0}{g}}^{\,0}}$$

$$= \frac{v_0 \sin\theta}{g} + \frac{v_0 \sin\theta}{g}$$

$$= \frac{2v_0 \sin\theta}{g} = \frac{2(2.51\,m/s)\sin(35°)}{9.81\,m/s^2}$$

$$= 0.29\,s$$

$$v_f = v_0 \cos\theta\,\hat{\imath} + (v_0 \sin\theta - gt)\hat{\jmath}$$

$$= (2.51\,m/s)\cos(35°)\hat{\imath} + ((2.51\,m/s)\sin(35°) - (9.81\,m/s)(0.29\,s))\hat{\imath}$$

$$= \boxed{2.06\,m/s\,\hat{\imath} - 1.41\,m/s\,\hat{\jmath}}$$