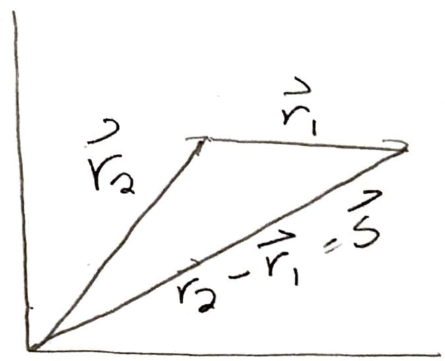
1/23/2018 LECTURE NOTES

IN ORDER TO UNDERSTAND MOTION WE MUST UNDERSTAND DISPLACEMENT. THERE IS LINEAR DISPLACEMENT AND ANGULAR DISPLACEMENT.

$\vec{S}$ = DISPLACEMENT

$\vec{d}$ = DISTANCE

LINEAR DISPLACEMENT



$$\vec{S} = \vec{r}_2 - \vec{r}_1 = \underbrace{(x_2 - x_1)\hat{i}}_{S_x} + \underbrace{(y_2 - y_1)\hat{j}}_{S_y} + \underbrace{(z_2 - z_1)\hat{k}}_{S_z}$$

$$|\vec{S}| = \sqrt{S_x^2 + S_y^2 + S_z^2}$$

$$\theta = \tan^{-1}\left(\frac{S_y}{S_x}\right)$$

$$\varphi = \tan^{-1}\left(\frac{S_z}{\sqrt{S_x^2 + S_y^2}}\right)$$

TAKE TH TWO VECTORS FROM 1/11/2018 LECTURE

$$\vec{r}_1 = 22.5\text{m}\,\hat{i} + 39.0\text{m}\,\hat{j}$$

$$\vec{r}_2 = 85.0\text{m}\,\hat{i} + 85.0\text{m}\,\hat{j}$$

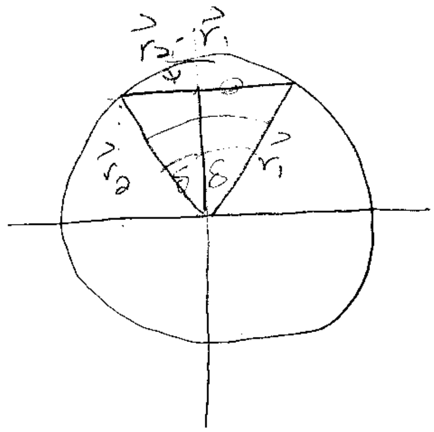
$$\vec{S} = (85.0 - 22.5)\text{m}\,\hat{i} + (85.0 - 39.0)\text{m}\,\hat{j}$$

$$= 62.5\text{m}\,\hat{i} + 46.0\text{m}\,\hat{j}$$

$$|\vec{S}| = \sqrt{(62.5\text{m})^2 + (46.0\text{m})^2} = 77.6\text{m}$$

$$\theta = \tan^{-1}\left(\frac{46.0\text{m}}{62.5\text{m}}\right) = 41.9^\circ$$

ANGULAR DISPLACEMENT



$$\text{ARC} = \ell = |\vec{r}|\,\theta = r\theta \qquad S, \theta = \text{RADIANS}$$

$$\vec{S} = \vec{r}_2 - \vec{r}_1 \Rightarrow |\vec{S}| = |\vec{r}_2 - \vec{r}_1|$$

$$\delta = \frac{\theta}{2}\ ; \quad \sin\delta = \frac{S/2}{r} = \frac{S}{2r}$$

$$r = |\vec{r}_1| = |\vec{r}_2|$$

FIND r, S, θ, ℓ

$$\vec{r}_1 = 12.5\text{m}\,\hat{i} + 21.7\text{m}\,\hat{j} \quad (\text{QUADRANT I})$$
$$\vec{r}_2 = -12.5\text{m}\,\hat{i} + 21.7\text{m}\,\hat{i} \quad (\text{QUADRANT II})$$

$$\vec{r}_1 = r = \sqrt{(12.5\text{m})^2 + (21.7\text{m})^2} = 25.0\text{m}$$

$$\vec{S} = (-12.5\text{m} - 12.5\text{m})\hat{i} + (21.7\text{m} - 21.7\text{m})\hat{j}$$
$$= -25.0\text{m}\,\hat{i}\ ; \quad |\vec{S}| = \sqrt{(-25.0\text{m})^2} = 25$$

$$\sin\delta = \frac{25.0\text{m}}{2(25.0\text{m})} = \frac{1}{2} \Rightarrow \delta = \sin^{-1}\left(\frac{1}{2}\right)$$
$$\delta = \frac{\pi}{6}$$

$$\theta = 2(\pi/6) = \pi/3$$

$$\ell = r\theta = (25.0\text{m})\,\pi/3 = 26.2\text{m}$$

VELOCITY

A RATIO OF DISTANCE TO TIME

- INSTANTANEOUS
- AVERAGE

UNITS (m/s)

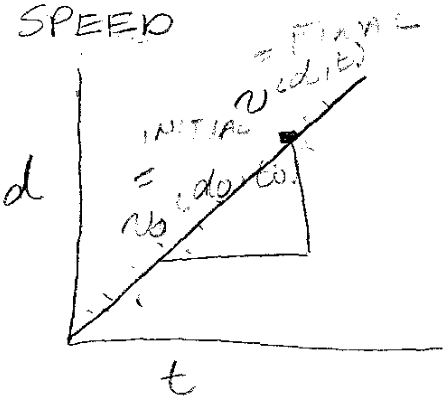
SPEED = MAGNITUDE

VELOCITY = MAGNITUDE + DIRECTIO

$\bar{v}$ = AVERAGE

$$\bar{v} = \frac{v_1 + v_2 + v_3 + v_5 + \dots + v_n}{n} = \frac{\sum_{i=1}^{n} v_n}{n} \leftarrow \text{SUM}$$

SPEED



$$\bar{v} = \frac{d - d_o}{t - t_o} \qquad t = t - t_o$$

$$t\,\bar{v} = \frac{d - d_o}{t}\,t \qquad \bar{v}t = d - d_o$$

$\boxed{\bar{v}t = d - d_o}$ AVERAGE

$\boxed{v t = d}$ INSTANTANEOUS

AS $d - d_o$ BECOMES SMALLER $\bar{v} = v$ OR INSTANTANEOUS

FIND $\bar{v}$ FOR FOLLOWING USING BOTH METHODS

t (s)	d (m)
1.0	3.0
2.0	4.0
3.0	9.0
4.0	15
5.0	25

v_1 = 3.0 m / 1.0 s = 3.0 m/s

v_2 = 4.0 m / 2.0 s = 2.0 m/s

v_3 = 9.0 m / 3.0 s = 3.0 m/s

v_4 = 15 m / 4.0 s = 3.8 m/s

v_5 = ~~25m / 5.0 s~~ = 5.0 m/s

AVERAGE = 3.4 m/s

$$\bar{v} = \frac{25\,m - 3.0\,m}{5.0\,s - 1.0\,s}$$

$\bar{v}$ = 5.5 m/s

LINEAR VELOCITY

$$\bar{\vec{v}} = \frac{\vec{s}}{t} = \frac{|\vec{r}_2 - \vec{r}_1|}{t} \qquad \bar{\vec{v}} = \frac{\vec{v} + \vec{v}_0}{2}$$

TAKE THE LINEAR DISPLACEMENT FROM ABOVE AND FIND THE AVERAGE VELOCITY IF $t = 4.20\,s$

$$\bar{\vec{v}} = \frac{\vec{s}}{t} = \frac{62.5m\,\hat{i} + 46.0m\,\hat{j}}{4.20s} = \frac{62.5m}{4.20s}\,\hat{i} + \frac{56.0m}{4.20s}\,\hat{j}$$

$$= \boxed{14.9\,m/s\,\hat{i} + 13.3\,m/s\,\hat{k}}$$

$$|\bar{\vec{v}}| = \sqrt{(14.9\,m/s)^2 + (13.3\,m/s)^2} = \bar{v}$$

$$\boxed{\bar{v} = 20.0\,m/s}$$

ANGULAR VELOCITY

$$\bar{v}\,t = d - d_0 = l$$

$$\frac{\bar{v}\,t}{t} = \frac{l}{t}$$

$$\bar{v} = \frac{l}{t} = \frac{r\theta}{t} \qquad \frac{\theta}{t} = \omega = \text{OMEGA}$$

$$\boxed{\bar{v} = r\omega}$$

WHAT IS AVERAGE VELOCITY FOR ANGULAR DISPLACEMENT IN THE SAME PERIOD

$$\bar{v} = \frac{r\theta}{t} = \frac{(25.0m)(\pi/3\ RAD)}{4.20s}$$

$$\boxed{\bar{v} = 6.23\,m/s}$$