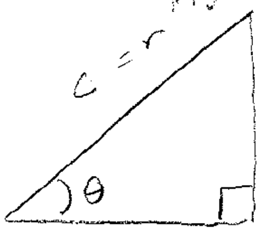
1/11/2018 LECTURE NOTES



PYTHAGOREAN THEORUM

$c^2 = a^2 + b^2$

$r^2 = x^2 + y^2$ OR $r = \sqrt{x^2 + y^2}$

MAGNITUDE

2 DIMENSIONAL

$\frac{y}{r} = \sin\theta$; $\frac{x}{r} = \cos\theta$; $\frac{y}{x} = \tan\theta$ ← DIRECTION

$y = r\sin\theta$ $\quad$ $x = r\cos\theta$

EXAMPLE

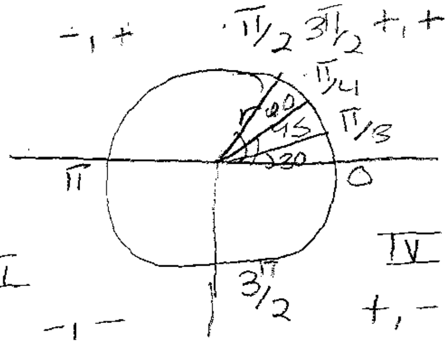
$\vec{r} = 45.0 \text{ km} \ @ \ \theta = 60.0°$

FIND THE COMPONENTS AND CONFIRM

$$\boxed{\begin{aligned} x &= (45.0\text{ km})\cos(60.0°) = 22.5\text{ km} \\ y &= (45.0\text{ km})\sin(60.0°) = 39.0\text{ km} \end{aligned}}$$

$\theta = \tan^{-1}\left(\frac{39.0\text{ km}}{22.5\text{ km}}\right) \sim 60.0°$ CONFIRMED

UNIT CIRCLE

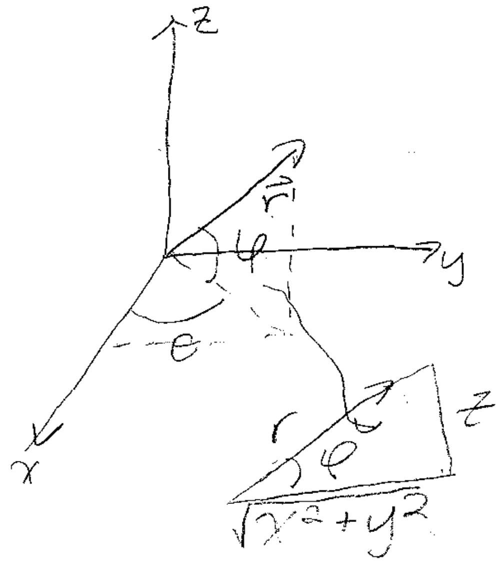


r = 1 UNIT

NEED TO CONFIRM MAGNITUDE AND DIRECTION

3-DIMENSIONAL

EUCLIDEAN (CUBICAL)



$x = r\cos\theta\cos\varphi$

$y = r\sin\theta\cos\varphi$

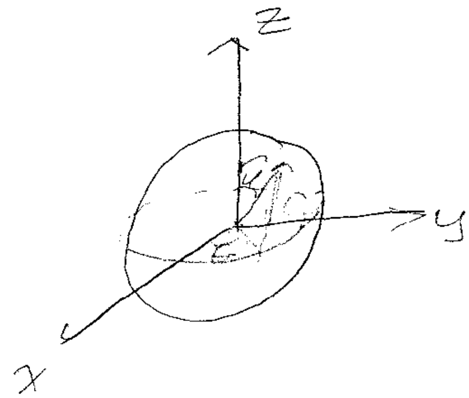
$z = r\sin\varphi$

$r = \sqrt{x^2+y^2+z^2}$

$\tan\theta = \dfrac{y}{x}$

$\tan\varphi = \dfrac{z}{\sqrt{x^2+y^2}}$

NON-EUCLIDEAN (SPHERICAL



$r = r$

$r\theta = r\theta$

$\varphi = r\cos\theta\,\varphi$

EXAMPLE

$\vec{r}_2 = 120\,m$ @ $\theta = 45.0°$; $\varphi = 20.0°$

FIND THE COORDINATES AND CONFIRM

$$\boxed{\begin{aligned} x &= (120\,m)\cos(45.0°)\cos(20.0°) = 79.7\,m \text{ OR } 0.0797\,km \\ y &= (120\,m)\sin(45.0°)\cos(20.0°) = 79.7\,m \text{ OR } 0.0797\,km \\ z &= (120\,m)\sin(20.0°) = 41.0\,m \text{ OR } 0.041\,km \end{aligned}}$$

$$\theta = \tan\left(\frac{79.7\,m}{79.7\,m}\right) = 45.0°$$

CONFIRMED

$$\varphi = \tan\left(\frac{41.0\,m}{\sqrt{(79.7\,m)^2 + (79.7\,m)^2}}\right) = 20.0°$$

ADDING VECTORS

- COMPONENTS
- GRAPHICALLY

$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

$$\vec{r}_1 + \vec{r}_2 = (x_1 + x_2)\hat{i} + (y_1 + y_2)\hat{j} + (z_1 + z_2)\hat{k}$$

EXAMPLE

ADD $\vec{r}_1$ AND $\vec{r}_2$

$$\vec{r}_1 = 45.0\,km\,\hat{i} + 39.0\,km\,\hat{j}$$

$$\vec{r}_2 = 0.0797\,km\,\hat{i} + 0.0797\,km\,\hat{j} + 0.041\,km\,\hat{k}$$

$$\vec{r}_1 + \vec{r}_2 = (45.0 + 0.797)\,km\,\hat{i} + (39.0 + 0.0797)\,km\,\hat{j} + (0 + 0.41)\,km\,\hat{k}$$

$$= \boxed{45.1\,km\,\hat{i} + 39.1\,km\,\hat{j} + 0.041\,km\,\hat{k}}$$

ADD GRAPHICALLY

$\vec{r}_1 = 45.0\,m \ @\ \theta = 60.0° = 22.5\,m\,\hat{i} + 39.0\,m\,\hat{j}$

$\vec{r}_2 = 120\,m \ @\ \theta = 45.0° = 85.0\,m\,\hat{i} + 85.0\,m\,\hat{j}$

$$\vec{r}_1 + \vec{r}_2 = (22.5 + 85.0)\,m\,\hat{i} + (39.0 + 85.0)\,\hat{j}$$
$$= 108\,m\,\hat{i} + 124\,m\,\hat{j}$$

$$|\vec{r}_1 + \vec{r}_2| = \sqrt{(108\,m)^2 + (124\,m)^2} = \boxed{164\,m}$$

$$\theta = \tan^{-1}\left(\frac{124\,m}{108\,m}\right) = \boxed{49.0°}$$